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Essays on monetary aspects of inflation

by

F. C. Nold
Jack L. Carr
John W. L. Winder

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by F. C. Nold

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INTEREST RATES AND INFLATION

IN CANADA AND THE UNITED STATES

by

F. C. Nold*

INTRODUCTION

The purpose of this study is to analyze the linear relationships among nominal interest rates in Canada and the United States and between these interest rates and intracountry inflation. Specifically, we will consider four topics. The first is the validity of viewing nominal interest rates as the sum of the real rate of interest, a liquidity premium and the expected rate of inflation calculated as a linear combination of actual rates of domestic inflation. The second topic is the so-called Gibson paradox, that is, the observed positive correlation between the level of prices and nominal interest rates. Both of these issues have been studied in the time domain using ordinary least squares,

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*I wish to thank J.F. McCollum, J.G. Cragg, and a reviewer for valuable comments and suggestions, and T. Amemiya and T.W. Anderson for introducing the author to the techniques used in this paper. I also take this opportunity to thank my wife, Ellen, for invaluable help in readying the manuscript for publication, and G. Suppes for the very able research assistance he provided. Of course, any errors are attributable to the author.

quasi-generalized least squares routines, and various distributed lag techniques in, for example, references [7], [10], [11], [12], [27], and [32]. The time domain analyses, while often encountering estimation difficulties in the form of autocorrelation or unbelievable lag structures, have generally been compatible with maintained economic doctrine. The third topic is the intranational term structure of interest rates. Finally, we examine the hypothesis that Canadian interest rates are largely determined by rates prevailing in the United States.

This study will concentrate on the frequency domain, using the techniques of spectral analysis. In general, spectral techniques are best suited to relatively long time series, usually containing two hundred or more observations. For this and other reasons, we shall use monthly data for the period January 1952 to July 1970. The advantages of this mode of analysis, which will become apparent as the analysis proceeds, come from a more efficient utilization of the information embodied in a time series.

This reclamation of information is purchased with the adoption of a stronger assumption concerning the behavior of the series in the time domain: covariance stationarity. While this assumption is, to a certain degree, implicit in much of the econometric work on these topics, we must recognize that many of the raw time series are not covariance stationary. We will cope formally with this difficulty (and as a by-product simplify the estimation problem) by prewhitening the time series with a judiciously chosen linear filter. The filter is presented in Appendix A, along with a detailed discussion of the time series used in this paper and an appraisal of the conformity of the prewhitened series to the stationarity assumption. Also, tests of hypotheses are made possible by assuming some subsets of the time series follow the multi-variate normal distribution. A partial test of this assumption is provided in the same appendix.

The rest of this paper is divided into six sections. The second section deals with the most rudimentary formulation of the hypothesized relationship between domestic rates of inflation and interest rates. The third analyzes this relationship in a multivariate context and can be viewed as the frequency domain analog of a reduced form equation. The fourth deals with the observed relationship between the level of domestic prices and domestic interest rates, exploring

both the bivariate and multivariate cases of this relationship. The fifth section investigates the term structure and also examines the association of Canadian interest rates with those in the United States, and a concluding section draws together the implications of this analysis for economic policy and for model building.

THE NAIVE FISHERIAN APPROACH

A great deal of attention has recently been directed towards the distinction between nominal and real interest rates. There can be no confusion about what the nominal interest rates are at any moment, but what is meant by real interest rates is often ill-defined. Irving Fisher [8] posited the relationship embodied in equation [1].

$$i_t^L = r_t^L + (\dot{P}/P)_t^{e,\,L} \qquad (1).$$

Where i_t^L is the nominal rate of interest on a bond of maturity, L at time t, r_t^L is the real rate of interest at time t for the period L, and $(\dot{P}/P)_t^{e,L}$ is the expected rate of price change at time t over the period L.

Equation (1) does not permit hypothesis testing as stated because it is either a tautology or an equilibrium condition. To provide a starting point, researchers have found it expedient to contend that expectations about inflation are formed from a stable linear function of past rates of inflation. Likewise, the crude assumption that r_t^L is a constant has often been adopted and the difficult problem of specifying the determinants of the real rate of interest have been thereby evaded. We shall adopt both of these contentions in this section, producing the naive Fisherian approach.

$$i_t^L = b_L + \sum_{s=0}^{n} a_s^L \, (\dot{P}/P)_{t-s} \qquad (2),$$

where b_L is a constant, $(\dot{P}/P)_{t-s}$ is the actual annual rate of price change in period t-s, and a_s^L is the weight given to period t-s in the formulation of price expectations applicable to the period L.

We have chosen to use the rate of change of the domestic

Consumer Price Index (CPI) to measure inflation because we are primarily interested in expectations. While it can be argued cogently that there are better measures of the nebulous quantity called the rate of inflation, the rate of change of the CPI is the index most often used for appraising the rate of movement in prices and presumably plays a prominent role in determining expectations about inflation.

One might be tempted to estimate the parameters of (2), using ordinary least squares after synthetically adding a supposedly independent, identically distributed error term. However, if ordinary least squares is used to estimate equation (2), three problems are encountered in general: a) multicollinearity of the lagged exogenous variables, b) serial correlation in the estimated residuals, and c) inadequate information about the appropriate specification of n. [1] Several distributed lag techniques could be used to deal with the first problem, if it occurs at all. The second problem, except in the case of prior knowledge about n, makes estimation quite difficult. If one knew n a priori, one could estimate equation (2) using ordinary least squares and compute a consistent estimate of the variance covariance matrix of residuals, Σ. One could then proceed with quasi-generalized least squares using the estimate of Σ.

All three difficulties may be avoided by making use of the properties of the variance covariance matrix of residuals and its relationship to spectral estimates. This mode of analysis was first suggested by E. J. Hannan [18] and can be interpreted in the time domain as an algorithm for generalized least squares. A summary of the procedure is presented in Appendix C.

The crux of the method is that we can obtain consistent estimates of the elements of the covariance matrix of residuals from the estimated spectral density of residuals.

In fact, the transformation back to an estimate of Σ is not necessary as we can solve a set of normal equations for our estimates of the parameters of equation (2). It is worth pointing out that, since we have left the lag structure virtually unspecified, this exercise is little more than an

---

[1] See Malinvaud, [24], chapter 15.

"uncritical description of the data".[2] Unambiguous conclusions can be drawn only when no relationship is found.

In bivariate regression analysis, testing correlation coefficients is equivalent to testing regression coefficients. In a similar way, it is instructive to look at the frequency domain analog of the correlation coefficient, the bivariate or marginal coherence function.[3] Table I contains a parametric summary statistic calculated from the estimated coherence between the nominal rate of interest on various bonds and the annual rate of inflation computed by extrapolating the month-to-month rate of change of the Consumer Price Index to an annual rate of inflation (denoted henceforth by MCPI).[4] Given the underlying multivariate normality of the time series, the summary statistic's asymptotic distribution is the standard normal and the test of no relationship is one-sided (since the coherence, by definition, is restricted[5] to [0,1]). We would reject the null hypothesis (no relationship) at the five per cent level if we had a value for the test statistic of 1.645 or greater.

The difference between the two test statistics presented for some of the entries in Table I is that the right-hand column entry is computed from estimates based on 37 lags while the left-hand entry is based on estimates using 48

---

[2] Hamon and Hannan [17], pg. 6039.

[3] This function is defined in Appendix B.

[4] This summary statistic is also discussed in Appendix B.

[5] It is not necessary to rely entirely on the test statistic derived in Appendix B. We can reduce our reliance on the assumption of multivariate normality of the underlying time series by using a non-parametric test, based on the Binomial distribution. The results of this non-parametric summary statistic are presented in Table B.1, (Appendix B). These results are included as additional evidence.

lags. [6]

The researcher must decide on how many lags he will use. A representative sample spectra is displayed in Figure 1. There are tradeoffs and subjective evaluations involved in choosing the number of lags to employ in estimation. Loosely stated, given a fixed sample size, the bias in estimation increases as the number of lags used decreases, and the variance increases as the number of lags used increases. This tradeoff relationship can be formalized into a series of uncertainty principles.[7] While we settled on 37 lags as a reasonable compromise for most of our work, estimates using 48 lags were often computed to see if key results were sensitive to the number of lags used. Although the summary statistic is biased upwards in both cases, the values based on 48 lags reflect more nearly independent estimates of the spectrums and cross-spectrums and are hence less biased.

Table II contains values for the same test statistic for the overall coherence between interest rates on bonds of various maturities and two slightly different indexes of price change. Since the behavior of MCPI from month to month is quite erratic, it is entirely possible that individuals use some sort of non-linear smoothing technique to extract the "true" pattern of price movements from the monthly observations. Of course, the number of conceivable candidates for such a smoothing device is unlimited and one could no doubt find some smoothing algorithm whose output covariates significantly with some nominal interest rate series. We restricted ourselves to two smoothing algorithms which are intuitively plausible: the three-month change in the price level extrapolated to an annual rate (QCPI) and the 12 month rate of change of the price level (YCPI). All the test statistics are based on spectral estimates, using 37 lags.

---

[6] Spectrum and cross-spectrum estimates can be produced by taking finite weighted sums of lagged covariances and cross-covariances. The number of lags denotes the number of such covariances included in the averaging procedure.

[7] See Dhrymes [4], p. 501.

TABLE I

Summary Test Statistics for the Relationship Between Interest Rates and Monthly Rates of Price Change

	U.S. MCPI		Canadian MCPI	
	48 Lags	37 Lags	48 Lags	37 Lags
Can. T.B.		-1.188	0.033	0.7403
Can. 1-3		-0.679	-0.364	-0.6157
Can. 3-5		0.859	-0.414	-0.3719
Can. 5-10		0.82	-0.145	0.2633
Can. 10+		-1.062	-0.294	-0.1618
McLeod Young Weir		0.003	0.3252	0.0127
U.S. T.B.	0.377	0.508		
U.S. 9-12 Mo.	0.245	0.291		
U.S. 3-5	0.282	0.45		
U.S. 10+	0.435	1.007		
Moody's	1.199	2.206		
Corporate	-0.101	0.009		
Canadian MCPI	3.549	4.31		

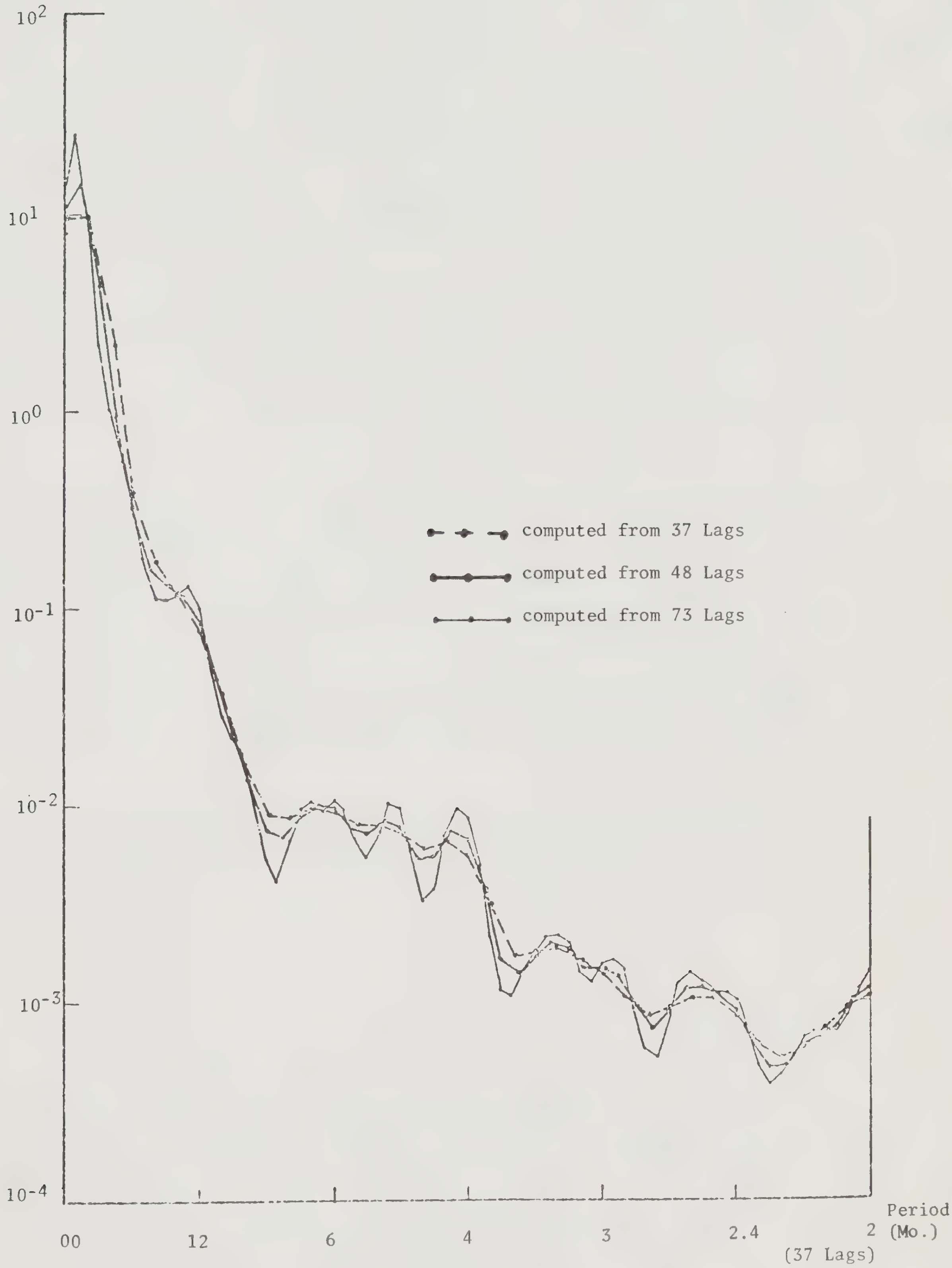
TABLE II

Summary Test Statistics for the Relationship Between Interest Rates and QCPI and YCPI

	Canadian		U.S.	
	QCPI	YCPI	QCPI	YCPI
Can. T.B.	0.207	-0.852	-0.444	-0.221
Can. 1-3	-0.429	0.748	0.188	-0.202
Can. 3-5	-0.809	-0.262	0.744	-0.339
Can. 5-10	-0.606	0.380	-0.510	0.082
Can. 10+	-0.524	0.963	-0.997	-1.560
McLeod Young Weir	0.118	1.00	0.164	0.983
Can. QCPI			7.612	2.102
Can. YCPI			3.536	6.912
U.S. T.B.			0.992	1.502
U.S. 9-12 Mo.			-0.449	-0.949
U.S. 3-5			0.582	-0.708
U.S. 10+			-0.354	0.086
Corporate			-0.759	-0.264
Moody's			-0.143	0.222

FIGURE 1

Sample Spectrum of
U.S. Treasury Bill Interest Rates
(Estimates Recolored)



The results in Tables I and II bring into question the existence of a significant linear relationship between the rate of inflation, however calculated, and nominal interest rates. We accept the null hypotheses (no relationship) for the association between interest rates and MCPI, QCPI and YCPI in every case. While it might be argued that the summary test statistic is deficient in that it does not recognize the significance of low frequency coherences vis à vis high frequency coherences given the "typical spectrum of an economic time series".[8] Figures 2 and 3 show that the individual coherences are generally insignificant.[9] Assuming the underlying time series are jointly normally distributed, only those points which lie above the dotted line are significant at the 10 per cent level. Conceivably, one could infer from the locations of the few significant coherences that proximate values of the rate of inflation have some effect on nominal interest rates. One must be careful, however, not to confuse covariation of two series with other time series for a genuine causal relationship. While this difficulty is ever present in econometrics, it is particularly acute when trend or seasonal patterns are involved.

We now turn to estimation of equation (2). There are several criteria available for choosing the appropriate value of n in the Hannan regression routine. However, the relationship embodied in equation (2) proved to be too weak to merit implementation of a full scale program of estimation. As a contrast to the estimates obtained from Hannan's routine, ordinary least squares and Durbin's two-step procedure, assuming first-order autocorrelation in the residuals, were also performed.

Before considering the estimated coefficients themselves, we will provide a heuristic explanation of how the Hannan

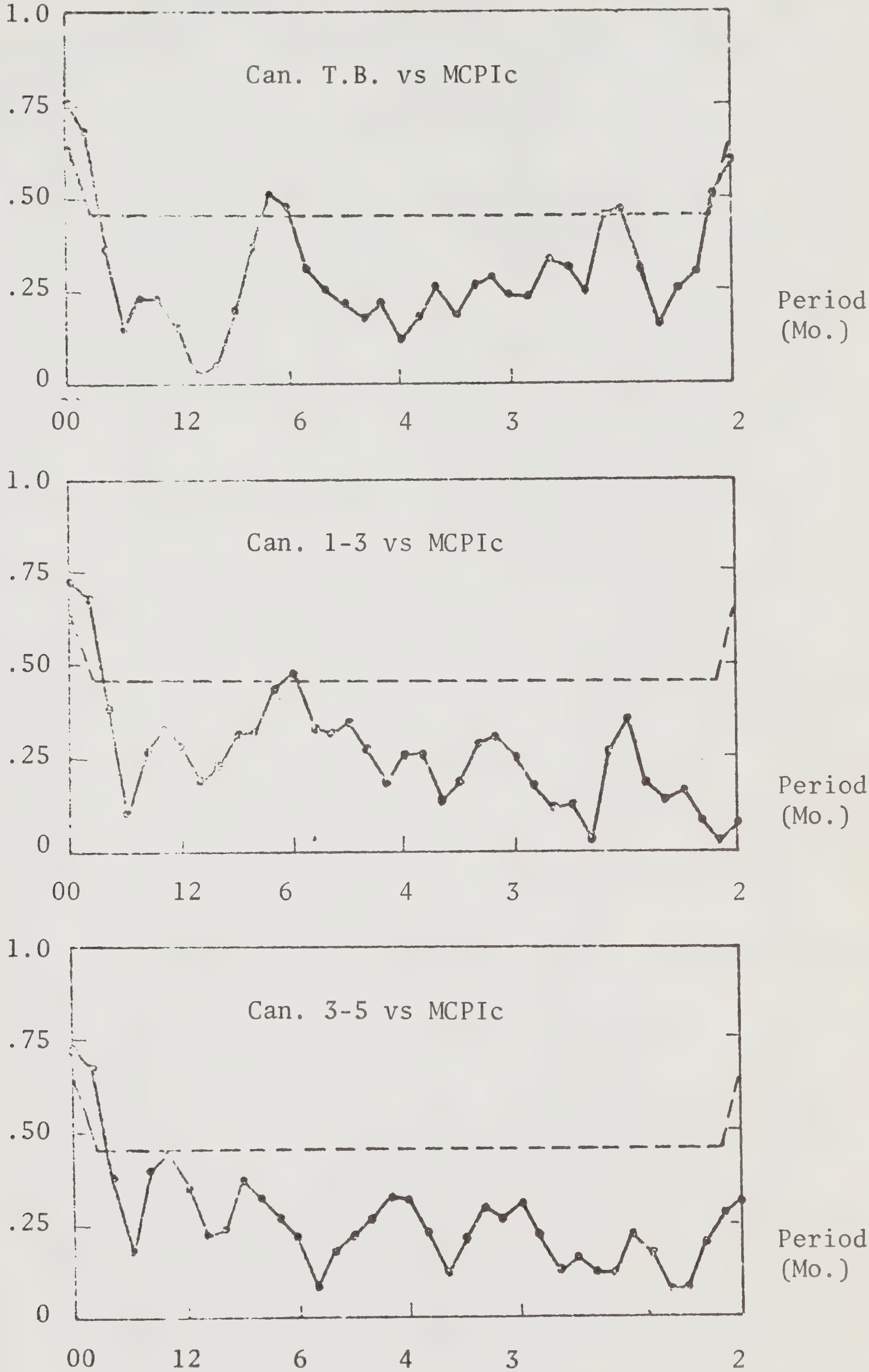
---

[8] See Granger [14].

[9] One interpretation of spectrum analysis is that it is a frequency domain variance decomposition. The low frequencies of economic time series generally contain a large proportion of the variance of the time series. Cohesion in these frequencies indicates that the factors which dominate the overall movements of the time series are related.

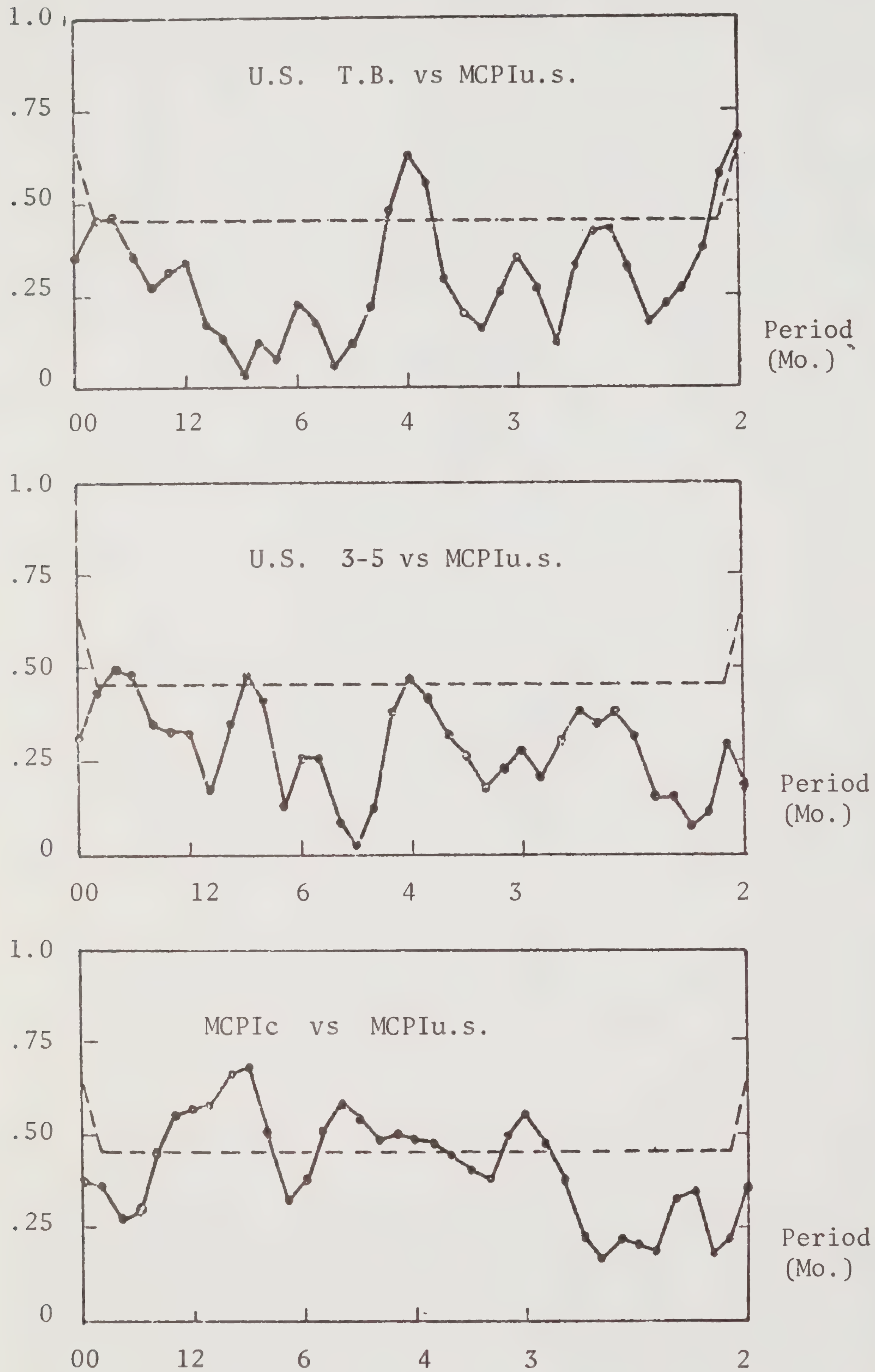
FIGURE 2

Coherence Functions for Naive Fisherian Hypothesis



MCPIc = Annual Rate of Canadian Inflation, (continued)

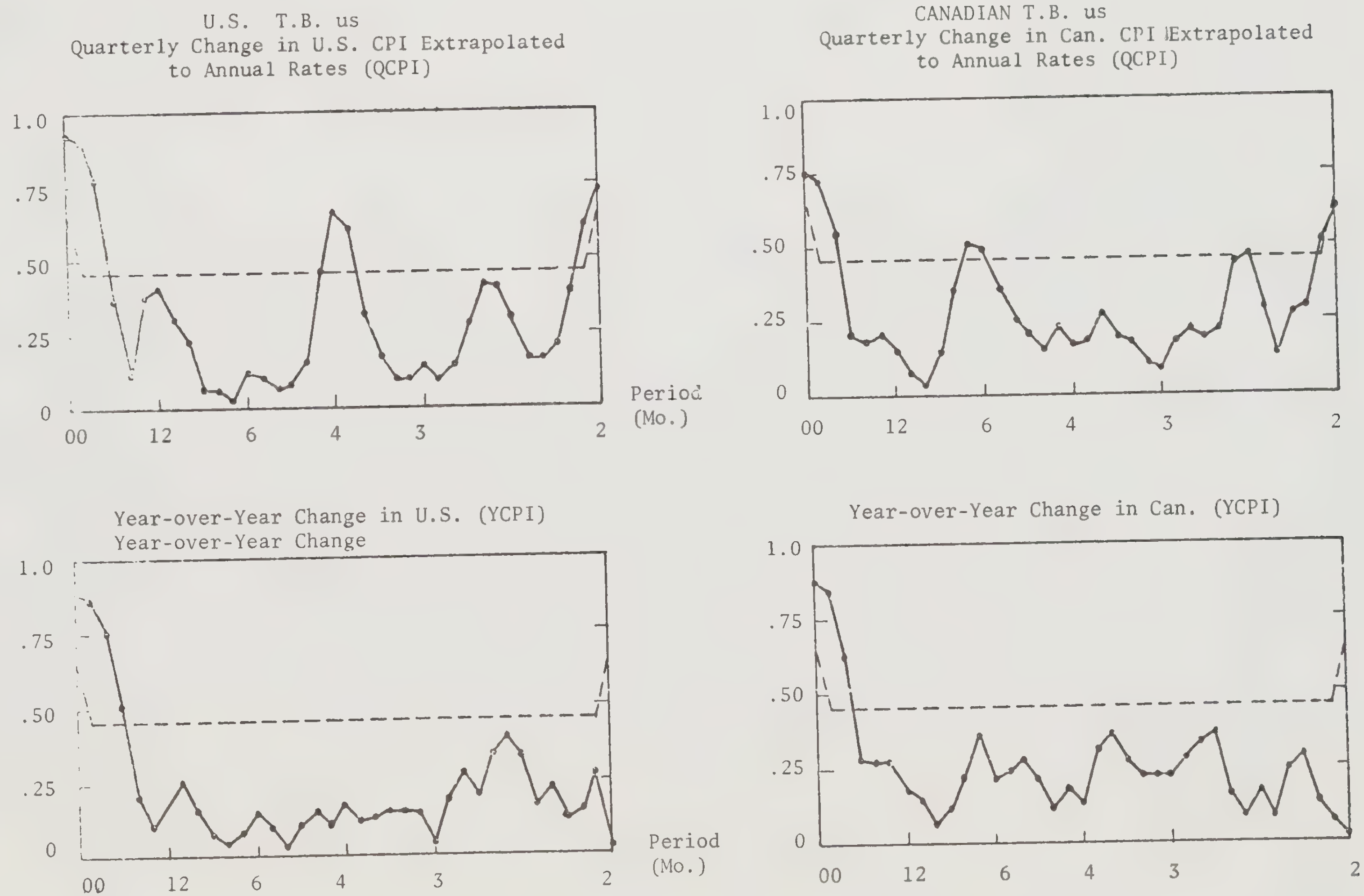
FIGURE 2 (continued)
Coherence Functions for Naive Fisherian Hypothesis



MCPIu.s. = Annual Rate of U.S. Inflation

FIGURE 3

Coherence: Under Different Types of Price Expectation Formulations



All Computations Based on 220 Observations and Spectrum Estimates Using 37 Lags

routine works, and consider the contexts in which it may have desirable properties. As a point of reference, we will consider the case where the standard assumptions for ordinary least squares are met. When we have independent, identically distributed residuals, the theoretical residual spectrum is a constant for all frequencies. (Its theoretical value is $\sigma^2/2\pi$.) Similarly, the theoretical signal-to-noise ratio has the same shape as the spectrum of the exogenous variable. The next simplest pattern for the residuals to follow is that of a first order Markov process. Given the usual positive autocorrelation ($1>\rho>0$) of economic time series, the theoretical residual spectrum declines monotonically as frequency increases. (Its theoretical values are $(1-2\rho\cos(2\pi/f) - \rho^2)\sigma^2/2\pi)$ where f stands for the frequency.) Also, given the typical shape of an economic time series, we might anticipate that the signal-to-noise ratio would be roughly constant. The residual spectrum and signal-to-noise ratio are defined in Appendix C.

Figures 4 and 5 are representative of the patterns of the sample residual spectrum and sample signal-to-noise ratio in the relationships we analyzed. The sample residual spectrum is not monotonic and contains too many spikes to even be approximated by a low-order Markov process. The sample signal-to-noise ratio varies by three orders of magnitude and generally behaves erratically. The characteristics of the residual spectrum and signal-to-noise ratios make estimation complicated and difficult, both in theory and in practice. It can be shown that the efficiency of ordinary least squares estimation with one exogenous variable has a lower bound of $4\lambda_S\lambda_L/(\lambda_S + \lambda_L)^2$, where λ_L and λ_S are the largest and smallest eigenvalues of Σ.[10] It can also be established that the maximum and minimum values of the spectral distribution function approximate the maximum and minimum eigenvalues of the variance covariance matrix. Hence, the lower bound of the efficiency of ordinary least squares in the case of esti-

---

[10]Additional but similar terms enter in the determination of the lower bound to the efficiency of the estimation routine as the number of lagged exogenous variables included in the relationship is increased. The lower bound to the efficiency will generally be a non-increasing function of the number of exogenous variables (because of the relationship between the arithmetic and geometric mean of two positive real numbers). See Watson [31].

FIGURE 4

Sample Residual Spectrum of Canadian Treasury Bills with Respect to MCPI

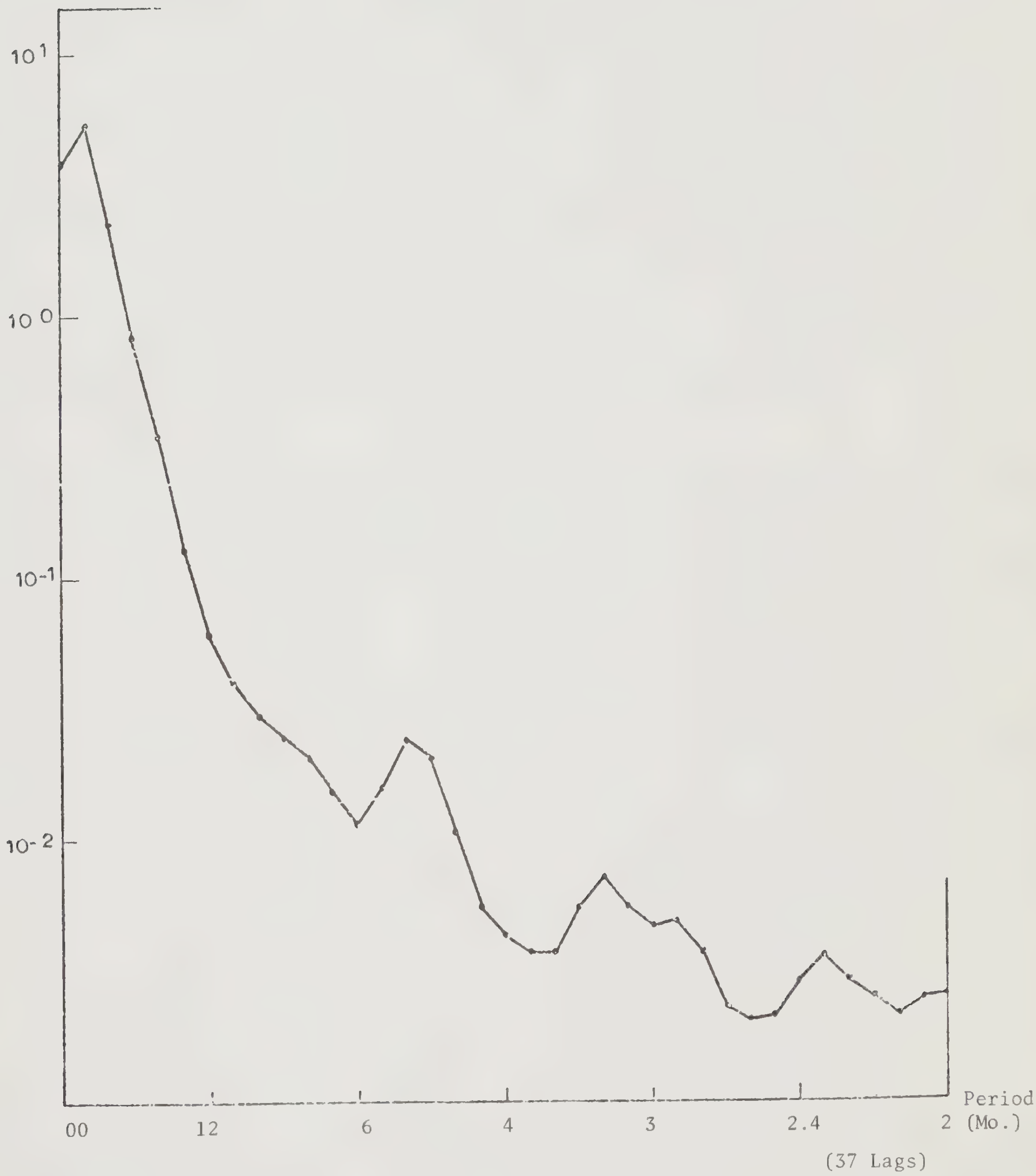
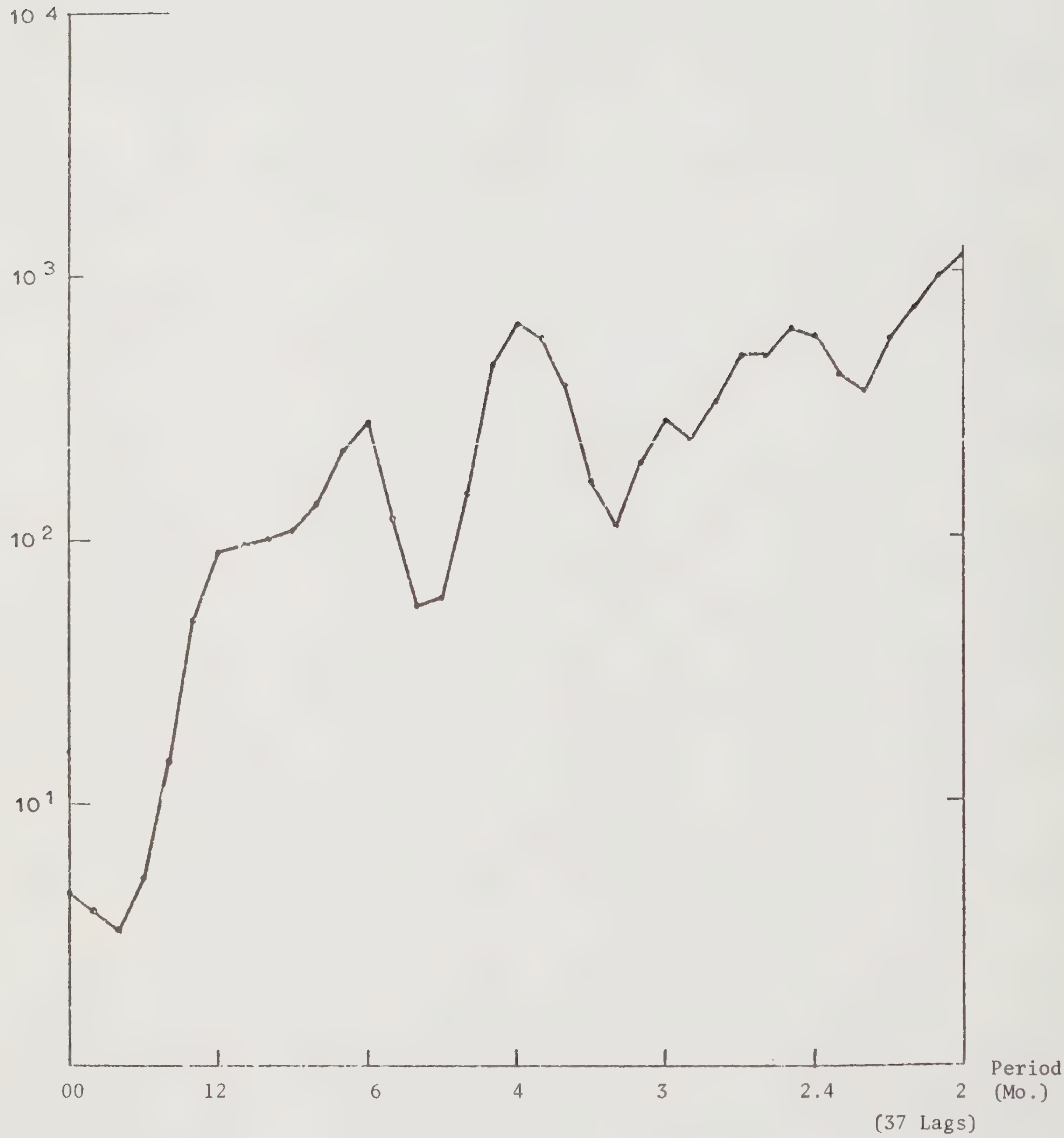


FIGURE 5

Sample Signal-To-Noise Ratio of Canadian Treasury Bills with Respect to MCPI



mating coefficients for the relationship between Canadian Treasury Bills and Canadian MCPI is approximately 0.0017. To put this lower bound into perspective, it would be the lower bound which would apply if we assumed the residuals in a bivariate regression problem were independent and identically distributed when they actually came from a first-order Markov process with the serial correlation coefficient equal to 0.96. Unfortunately, there is no simple way to approximate the lower bound to the efficiency of the Durbin two-step procedure. While this approach takes account of the general shape of the spectrum of residuals, it cannot conform to its actual complicated behavior.

Table III contains the Hannan estimates for the coefficients when zero, three, six, twelve and eighteen lagged values of the domestic rates of inflation are used as the dependent variables. Similar results were obtained for the other interest rate series and estimated coefficients for relationships with up to 36 lagged rates of domestic inflation. These are not reproduced here in the interest of brevity. Representative examples of identical lag structures estimated with other procedures are given in Table IV.

We anticipate that the weights in the formulation of the expected rate of inflation are going to be positive. Hence, the critical regions will be one-sided. The asymptotic distribution of the coefficients estimated with Hannan's routine is normal. Testing the null hypothesis ($a_s^L \leq 0$) at the five per cent level, we can reject it for only three coefficients of the 44 estimated in the Canadian case and two in the U.S. case when we use MCPI as the independent variable. No coefficient in either case is significant at the one per cent level. Both total numbers of significant coefficients at the five per cent level are well within the range of expected totals generated randomly if the null hypothesis were, in fact, true. When QCPI is used as the independent variable, the estimated coefficients are somewhat more likely to be significant in the U.S. case while remaining generally insignificant in the Canadian case. Seven of the 44 coefficients presented are significant at the one per cent level in the results using the U.S. QCPI. It should be noted, however, that all the estimated coefficients are quite small in magnitude. It is only when we use YCPI as the exogenous variable that large numbers of coefficients fall into the rejection region and have reasonably large magnitudes. A rationale for the results when YCPI is used is given in the

next section.

The results from the ordinary least squares analysis and, to a certain extent, Durbin's two-step procedure convey a considerably different picture. The coefficients estimated by ordinary least squares using domestic MCPI as the exogenous variable strongly support the Naive Fisherian relationship for both the U.S. and Canadian data. The results from the Durbin two-step procedure are somewhat ambiguous, tending on the whole to give some support to the hypothesized relationship.

The findings of this section may be summarized as follows. We have looked at the sample marginal coherence between interest rates and rates of domestic inflation. Two approaches to this data were adopted, testing the summary statistic and testing the individual coherences. Neither lent support to the Naive Fisherian hypothesis. We also translated our spectral estimates back into the time domain. Unambiguous and negative results were obtained with the Hannan estimates for the case in which the rate of inflation was calculated by extrapolating month-to-month changes in the CPI. However, the other more stable measures of annual inflation gave ambiguous results with some support for the Naive Fisherian hypothesis discernable when YCPI was used as the exogenous variable. The ideal situation for studying the effect of price expectations on nominal interest rates would present the researcher with sufficient data to represent both the supply and demand sides of the bond market as well as some more reasonable (non-linear) way of assessing the price expectations of lenders and borrowers. As this ideal is not attainable, we must be satisfied with whatever insight we can glean from rudimentary formulations of the Fisherian hypothesis. It is in this spirit that the analysis is extended to the more realistic, but still deficient, single equation multivariate regression model.

FISHER EFFECTS IN A MULTIVARIATE CONTEXT

In this section, we will relax the assumption that r_t is a constant. The approach will be, by necessity, analogous to the treatment of reduced form, single equation models common in this area of empirical research. Generally speaking, any variable vaguely associated with the determination of either the supply of or demand for bonds is fair game for inclusion

TABLE III

(a) Canadian T.B. As A Linear Function of Hannan Estimates.

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
0	0	0.398	-3	0.180	-1	0.962	-1
		(0.099)		(1.665)		(1.710)	
3	0	-0.803	-2	0.891	-2	0.815	-1
		(-1.559)		(0.696)		(1.438)	
	1	-0.130	-1	-0.158	-1	0.149	
		(-2.143)		(-1.452)		(2.633)	
	2	-0.666	-2	-0.425	-2	0.153	
		(-1.098)		(-0.392)		(2.714)	
	3	-0.118	-1	-0.167	-1	0.227	-1
		(-2.281)		(-1.300)		(0.401)	
6	0	-0.716	-2	0.100	-1	0.911	-1
		(-1.304)		(0.727)		(1.593)	

TABLE III(a), (continued)

	1	-0.913 (-1.307)	-2	-0.324 (-0.024)	-3	0.166 (2.909	
	2	-0.284 (0.378)	-2	-0.732 (-0.561)	-2	0.151 (2.646)	
	3	-0.527 (-0.656)	-2	-0.720 (-0.451)	-2	0.154 (0.271)	-1
	4	0.837 (1.115)	-2	0.226 (1.731)	-1	0.113 (1.982)	
	5	-0.164 (-0.234)	-2	-0.612 (-0.454)	-2	0.868 (1.524)	-1
	6	0.829 (-0.151)	-3	0.159 (1.150)	-1	0.462 (0.809)	-1
12	0	-0.527 (-0.871)	-2	0.169 (1.055)	-1	0.873 (1.435)	-1
	1	-0.479 (-0.579)	-2	0.154 (0.096)	-2	0.147 (2.431)	

Rejection region for a one-tailed test of H_O, $A_S^L \leq 0$, at the five per cent level is test statistic >1.645. The test statistic appears in parentheses under each estimated coefficient.

12	2	0.332	-2	-0.543	-2	0.154	
		(0.344)		(-0.329)		(2.541)	
	3	0.292	-2	0.655	-2	0.316	-1
		(0.266)		(0.304)		(0.525)	
	4	0.182	-1	0.281	-1	0.109	
		(1.551)		(1.382)		(1.832)	
	5	0.101	-1	0.282	-3	0.682	-1
		(0.803)		(-0.013)		(1.165)	
	6	0.145	-1	0.356	-1	0.391	-1
		(1.115)		(1.526)		(0.676)	
	7	-0.143	-1	0.298	-2	0.742	-1
		(1.134)		(0.141)		(1.266)	
	8	0.128	-1	0.780	-2	0.255	-1
		(1.091)		(0.384)		(0.427)	
	9	0.141	-1	0.237	-1	0.192	-1
		(1.286)		(1.098)		(0.319)	

TABLE III(a), (continued)

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
12	10	0.119	-1	0.319	-2	-0.414	-1
		(1.233)		(0.193)		(-0.681)	
	11	0.847	-2	0.679	-2	0.528	-2
		(1.025)		(0.421)		(0.087)	
	12	0.510	-2	0.896	-2	0.539	-1
		(0.842)		(0.560)		(0.886)	
18	0	-0.530	-2	0.166	-1	0.760	-1
		(-0.866)		(1.020)		(1.216)	
	1	-0.564	-2	0.200	-2	0.153	
		(-0.673)		(0.122)		(2.447)	
	2	0.327	-2	-0.633	-2	0.147	
		(0.336)		(-0.378)		(2.377)	
	3	0.376	-2	0.614	-2	0.178	-1
		(0.337)		(0.283)		(0.287)	
	4	0.201	-1	0.329	-1	0.122	
		(1.646)		(1.499)		(1.956)	

18	5	0.136	-1	-0.122	-2	0.483	-1
		(1.027)		(-0.053)		(0.776)	
	6	0.204	-1	0.406	-1	0.336	-1
		(1.447)		(1.616)		(0.546)	
	7	0.221	-1	0.130	-1	0.742	-1
		(1.534)		(0.517)		(1.201)	
	8	0.223	-1	0.104	-1	0.184	-1
		(1.536)		(0.412)		(0.290)	
	9	0.262	-1	0.341	-1	0.741	-2
		(1.792)		(1.284)		(1.117)	
	10	0.252	-1	0.191	-1	-0.437	-1
		(1.740)		(0.757)		(-0.689)	
	11	0.221	-1	0.101	-1	0.888	-5
		(1.539)		(0.401)		(0.000)	
	12	0.206	-1	0.233	-1	0.583	-1
		(1.464)		(0.928)		(0.947)	

TABLE III(a), (continued)

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
18	13	0.160 (1.209)	-1	0.177 (0.768)	-1	0.581 (0.933)	-1
	14	0.106 (0.863)	-1	0.109 (0.005)	-3	-0.305 (-0.490)	-1
	15	0.120 (1.073)	-1	0.166 (0.764)	-1	0.235 (0.379)	-1
	16	0.682 (0.700)	-2	0.622 (0.372)	-2	0.457 (0.739)	-1
	17	0.243 (0.290)	-2	-0.560 (-0.003)	-4	0.203 (0.325)	-1
	18	0.367 (0.600)	-2	0.631 (0.388)	-2	0.201 (0.322)	-1

TABLE III

(b) United States T.B. As A Linear Function of Hannan Estimates.

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
0	0	0.562	-2	-0.173	-1	0.113	+0
		(1.748)		(-1.689)		(2.403)	
3	0	0.600	-2	-0.520	-2	0.121	
		(1.100)		(-0.387)		(2.473)	
	1	0.244	-2	0.358	-3	0.140	
		(0.346)		(0.029)		(2.858)	
	2	-0.462	-2	0.150	1	0.148	
		(-0.655)		(1.209)		(3.006)	
	3	-0.106	-1	0.297	-1	0.145	
		(-2.081)		(2.206)		(2.980)	
6	0	0.116	-2	0.298	-2	0.117	
		(1.492)		(0.209)		(2.351)	
	1	0.145	-2	-0.402	-3	0.125	
		(0.985)		(-0.028)		(2.507)	

TABLE III(b) (continued)

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
6	2	0.235 (0.284)	-2	0.304 (2.139)	-1	0.140 (2.771)	
	3	0.829 (0.906)	-3	0.401 (2.374)	-1	0.132 (2.638)	
	4	0.133 (1.604)	-1	-0.618 (-0.435)	-2	0.320 (0.637)	-1
	5	0.927 (1.225)	-2	0.330 (2.286)	-1	0.118 (2.378)	
	6	0.567 (1.090)	-2	0.224 (1.571)	-1	0.501 (1.010)	-1
12	0	0.625 (1.164)	-2	0.574 (0.389)	-2	0.125 (2.355)	
	1	0.456 (0.566)	-2	0.155 (0.998)	-1	0.125 (2.410)	
	2	-0.630 (0.069)	-3	0.363 (2.339)	-1	0.148 (2.814)	

Rejection region for a one-tailed test of H_o, $A_S^L \leq 0$, at the five percent level is test statistic > 1.645. The test statistic appears in parentheses under each estimated coefficient.

12	3	-0.295 (0.302)	-2	0.506 (2.691)	-1	0.139 (2.655)	
	4	0.145 (0.740)	-2	0.291 (1.563)	-1	0.221 (0.415)	-1
	5	0.206 (0.198)	-2	0.387 (2.059)	-1	0.132 (2.488)	
	6	-0.299 (-0.283)	-2	0.493 2.464)	-1	0.331 (0.641)	-1
	7	0.903 (-0.869)	-2	0.487 (2.594)	-1	-0.227 (-0.428)	-1
	8	0.341 (-0.338)	-2	0.125 (0.067)	-2	0.130 (0.245)	-1
	9	-0.595 (-0.610)	-2	0.255 (1.355)	-1	-0.508 (-0.967)	-1
	10	-0.920 (-1.076)	-2	0.234 (1.508)	-1	0.166 (0.317)	-1

TABLE III(b), (continued)

No. of Lags		M CPI Coefficient		Q CPI Coefficient		Y CPI Coefficient	
12	11	-0.298	-2	-0.188	-2	-0.502	-2
		(-0.369)		(-0.121)		(-0.097)	
	12	-0.663	-3	-0.108	-2	0.147	-1
		(-0.123)		(-0.073)		(0.277)	
18	0	0.608	-2	0.582	-2	0.111	
		(1.110)		(0.392)		(1.995)	
	1	0.435	-2	0.152	-1	0.114	
		(0.526)		(0.977)		(2.079)	
	2	-0.133	-2	0.350	-1	0.149	
		(0.140)		(2.245)		(2.701)	
	3	-0.345	-2	0.507	-1	0.139	
		(-0.334)		(2.669)		(2.519)	
	4	0.737	-2	0.280	-1	0.239	-1
		(0.682)		(1.469)		(0.427)	
	5	0.195	-2	0.374	-1	0.138	
		(0.172)		(1.944)		(2.468)	
	6	-0.281	-2	0.513	-1	0.433	-1
		(-0.239)		(2.504)		(0.772)	

Table IV

RESULTS FOR ORDINARY LEAST SQUARES AND DURBIN'S TWO-STEP PROCEDURE FOR TREASURY BILLS REGRESSED ON DOMESTIC MCPI

	Canadian		United States	
Number of Lags	Ordinary Least Squares Coefficient	Durbin 2 Step Coefficient	Ordinary Least Squares Coefficient	Durbin 2 Step Coefficient
0	0.122	0.001	0.305	-0.005
	(3.937)	(-0.232)	(8.797)	(-1.067)
3	0.103	0.002	0.159	-0.007
	(3.515)	(0.428)	(4.983)	(-1.267)
	0.072	0.008	0.125	0.006
	(2.408)	(1.235)	(3.828)	(-1.013)
	0.078	0.013	0.128	0.001
	(2.609)	(2.098)	(3.941)	(0.128)
	0.115	0.006	0.167	0.006
	(3.929)	(1.038)	(5.301)	(1.050)
6	0.100	0.004	0.110	-0.006
	(3.819)	(0.655)	(3.706)	(-1.112)
	0.085	0.010	0.086	-0.008
	(3.129)	(1.463)	(2.868)	(-1.155)
	0.077	0.015	0.078	-0.001
	(2.86)	(2.164)	(2.591)	(0.100)
	0.096	0.010	0.115	0.003
	(3.567)	(1.344)	(3.905)	(0.401)
	0.067	0.004	0.084	-0.006
	(2.457)	(0.583)	(2.833)	(-0.862)
	0.080	0.001	0.101	0.006
	(-2.973)	(0.191)	(3.462)	(0.925)
	0.119	0.007	0.114	0.005
	(4.57)	(1.237)	(3.966)	(0.809)

Rejection region for a one-tailed test of H_0, $A_S^L \leq 0$, at the five per cent level is test statistic > 1.645. The test statistic appears in parentheses under each estimated coefficient.

TABLE IV (continued)

Number of Lags	Canadian Ordinary Least Squares Coefficient	Canadian Durbin 2 Step Coefficient	United States Ordinary Least Squares Coefficient	United States Durbin 2 Step Coefficient
12	0.053 (2.19)	0.005 (0.757)	0.095 (3.136)	-0.002 (-2.91)
	0.067 (2.883)	0.013 (1.828)	0.079 (2.641)	0.000 (0.057)
	0.080 (3.459)	0.020 (2.626)	0.071 (2.366)	0.012 (1.427)
	0.082 (3.552)	0.017 (2.059)	0.091 (3.047)	0.019 (2.017)
	0.073 (3.118)	0.020 (1.438)	0.082 (2.786)	0.013 (1.373)
	0.079 (3.421)	0.011 (1.229)	0.082 (2.815)	0.027 (2.753)
	0.086 (3.749)	0.018 (2.059)	0.084 (2.890)	0.027 (2.749)
	0.082 (3.561)	0.015 (1.714)	0.083 (2.848)	0.026 (2.725)
	0.077 (3.316)	0.019 (2.335)	0.054 (1.814)	0.023 (2.528)
	0.070 (3.017)	0.012 (1.439)	0.027 (0.924)	0.018 (2.121)
	0.059 (2.552)	0.006 (0.807)	0.019 (0.650)	0.013 (1.814)
	0.046 (2.003)	-0.004 (-0.550)	0.006 (0.210)	0.009 (1.413)
	0.045 (1.903)	0.001 (0.215)	-0.037 (-1.266)	0.001 (-0.277)
18	0.048 (2.006)	0.005 (0.727)	0.094 (3.012)	-0.004 (-0.696)
	0.051 (2.118)	0.013 (1.733)	0.086 (2.717)	-0.000 (-0.027)

TABLE IV (continued)

Number of Lags	Canadian Ordinary Least Squares Coefficient	Canadian Durbin 2 Step Coefficient	United States Ordinary Least Squares Coefficient	United States Durbin 2 Step Coefficient
18	0.063 (2.6)	0.021 (2.684)	0.067 (2.121)	0.011 (1.317)
	0.067 (2.727)	0.017 (2.041)	0.087 (2.732)	0.016 (1.731)
	0.061 (2.459)	0.013 (1.494)	0.088 (2.824)	0.012 (1.270)
	0.063 (2.541)	0.011 (1.257)	0.081 (2.608)	0.024 (2.403)
	0.069 (2.811)	0.019 (2.030)	0.074 (2.407)	0.022 (2.079)
	0.074 (3.148)	0.017 (1.754)	0.081 (2.694)	0.017 (1.690)
	0.074 (3.215)	0.022 (2.300)	0.048 (1.597)	0.011 (1.126)
	0.067 (2.873)	0.014 (1.516)	0.024 (0.802)	0.005 (0.574)
	0.060 (2.595)	0.009 (0.913)	0.015 (0.491)	-0.001 (-0.78)
	0.046 (1.95)	-0.001 (-0.078)	0.011 (0.351)	-0.006 (-0.664)
	0.039 (1.609)	0.005 (0.584)	-0.032 (-1.033)	-0.015 (-1.789)
	0.037 (1.535)	0.005 (0.611)	-0.028 (-0.905)	-0.018 (-2.312)
	0.025 (1.026)	0.001 (0.065)	-0.001 (-0.027)	-0.016 (-2.102)
	0.024 (0.997)	0.003 (0.377)	0.007 (0.213)	-0.009 (-1.217)
	0.017 (0.683)	0.001 (0.112)	-0.005 (-0.171)	-0.014 (-2.022)

TABLE IV (continued)

	Canadian		United States	
Number of Lags	Ordinary Least Squares Coefficient	Durbin 2 Step Coefficient	Ordinary Least Squares Coefficient	Durbin 2 Step Coefficient
18	0.018	0.001	0.017	-0.003
	(0.739)	(0.214)	(0.565)	(-0.511)
	0.027	0.004	0.039	0.002
	(1.113)	(0.624)	(1.322)	(0.335)

in a reduced form equation.[11]

Several factors limit the number of exogenous variables which we can include in the relationship in the present analysis. First, only a few economic time series are compiled monthly so as to be compatible with the series on bond rates and rates of inflation. Second, the programming difficulties increase quite rapidly as more variables are included in a relationship because we have to invert and manipulate Hermitian matrices having the same order as the number of exogenous variables. Finally, our ability to discriminate between the null hypothesis and various alternatives is rapidly eroded by inclusion of more exogenous variables. This problem arises because, given the number of lags used in the estimation phase, the number of observations and the lag window employed, the analysis starts with only 11 effective degrees of freedom (E.D.F.). The inclusion of each additional exogenous variable diminishes the degrees of freedom applicable to conditional statistics by one. Given these limitations, two approaches are feasible and both have been explored. One approach is to include those variables which are most important on theoretical or empirical grounds. Another approach is to extract principle components from the set of possible independent variables or from certain subclasses of these variables.

[11]See references [7], [10], [11], [12], [27], and [32].

The class of time domain models we will consider in this section, with the exception of the results using principal components as two of the independent variables, are those generated by:

$$r_t = b + \sum_{s=N_1}^{N_2} a_s(\Delta P/P)_{t-s} + \sum_{v=N_3}^{N_4} d_v RPCG_{t-v} + \sum_{u=N_5}^{N_6} c_u RPCM_{t-u} \quad (3)$$

where RPCM is real, per capita M_2 and RPCG is real, per capita Gross National Product.[12] The superscript indicating maturity, L, is suppressed to keep the notation manageable, and we leave the N's, a's, d's and c's virtually unspecified, requiring only that they be real constants.

The analysis in this section is based on several types of coherence functions. We shall require a measure of that part of the association between two time series which is independent of (orthogonal to) the covariation generated by mutual relationships with other time series.[13] This role is performed by the partial correlation coefficient in the time domain, while in the frequency domain the role is filled by the partial coherence function. We also need to appraise the strength of the overall relationship between interest rates and their hypothesized determinants. This is accomplished by the multiple correlation coefficient in the time domain. The quantity we use in this analysis is the multiple coherence function.

There are two reasons why the multivariate time-domain analysis is inferior to the frequency-domain approach adopted in this paper. Besides the problem of specifying the determinants of interest rates, we must cope with the problem of specifying the lag structure for each of these exogenous variables. Without such a priori specification, the number of feasible and reasonable time-domain models is large. Indeed, no optimal procedure exists for sorting through the models, and we would be forced to fall back to ad hoc proce-

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12 The derivations of proxies for RPCG and RPCM are discussed in Table A.1 of Appendix A.

13 A more detailed discussion of these functions can be found in several books and articles [1], [4], [9], [13], [15] and in Appendix B.

dures. However, as long as we can estimate the relevant spectrums and cross-spectrums with accuracy, we can test all such time-domain models simultaneously against the null hypothesis that none of these models are adequate using the invariance property of the coherence functions. We are able to accept or reject a whole class of models or to decide whether a specific variable contains any significant, independent explanatory power.[14] On the other hand, we cannot discriminate at all between different variants from the class of models which contains a particular set of exogenous variables.

The second advantage of the frequency-domain approach is that we can examine the covariation of the series at each frequency. This is of value when checking the validity of a model by seeing if some important aspects of the variation of the dependent variable have been left unexplained (using the relative values of the spectrum of the dependent variable as the criteria of importance). More important for the present analysis is the ability to see what the introduction of other time series does to the partial coherence function in each frequency band.

This second advantage to frequency-domain analysis is, however, not without its drawbacks. While we can test each estimated coherence for significance, we again need a method to summarize the information embodied in the coherence function to get a crude idea of the overall strength of the relationship. The test statistic in the previous section can be used here with some minor modifications which are explained in Appendix B.[15]

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[14] If interest rates respond at all to the levels of RPCG and RPCM, they probably respond to movements which are viewed as permanent. As long as the permanent components are extracted from the monthly values by linear operations, we can use the invariance property of the coherence functions to test the whole group of variables. A brief discussion of the invariance of coherence function is provided in Appendix B.

[15] The qualifications outlined in the previous section apply here also. Relevant supporting evidence obtained from the non-parametric test mentioned in the previous section and discussed in Appendix B is presented in Table B.1.

The summary test statistics for the multivariate analyses are presented in Table V. Several features of the results are obvious. First, the test statistics for the multiple coherence functions are generally large and positive for the U.S. series. However, no similarly strong association emerges for the corresponding Canadian series. This finding indicates that a considerably stronger relationship exists between the various combinations of exogenous variables and interest rates in the U.S. than in Canada. This is a predictable result because we expect a strong tendency for Canadian rates to follow closely movements of the rates on U.S. bonds of similar maturity. This tendency is explored in the fifth section of this study. Second, the summary test statistics for the partial coherence between various indicators of the rate of domestic inflation and interest rates are negative and relatively large in magnitude. The effect of conditionalizing the impact of the rate of domestic inflation on interest rates is the eradication of an already weak relationship.

The partial coherence functions between domestic MCPI and selected representative interest rate series conditionalized on RPCG and RPCM are presented in Figure 6. Figure 7 contains the complementary multiple coherence functions. We would expect the partial coherence function to have randomly two points above the dotted line, which indicates the boundary of the critical region for testing the null hypothesis at the 10 per cent level, even if the null hypothesis were true.[16] It is remarkable that none of the partial coherence functions in Figure 6 has as many points as expected in the critical region. Since results for the non-parametric test for the multiple coherence functions when Canadian interest rates are the dependent variables corroborate the conclusions drawn from the parametric summary statistics, we will restrict the rest of the discussion in this section to results for U.S. series.

When the smoothed measures of inflation, QCPI and YCPI, are used in combination with RPCG and RPCM, the summary test

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[16] If every other point at which the coherence function is estimated is taken as an independent observation on a random variable having the binomial distribution with P = 0.1, the expected number of points above the dotted line is 1.9 in our case.

TABLE V

Multivariate Summary Statistics

	I		II	
	Partial	Multiple	Partial	Multiple
U.S. Bonds:				
Treasury Bill	-2.034	2.285	-1.903	1.747
9-12 months	-3.049	2.035		
3-5 years	-1.731	1.26	-1.86	0.746
10+ years	-1.997	1.26	-1.874	0.937
Corporate	-2.698	0.717		
Moody's	-0.572	2.814		
Canadian Bonds:				
Treasury Bill	-3.709	-1.451	-3.227	-1.933
1-3 years				
3-5 years	-3.67	-2.785	-3.437	-3.02
5-10 years				
10+ years	-2.548	-2.224	-2.363	-1.193
McLeod, Young, Weir	-2.041	-2.44		
One Tail Test, Rejection of H_0 (no relationship) is Test Statistic >1.645.	Domestic or MCPI conditionalized and in combination with domestic RPCG and RPCM.		Domestic MCPI conditionalized and in combination with the first two principle components, chosen from RPCG, RPCM, and changes in RPCG and RPCM.	

TABLE V

(continued)

	III		IV	
	Partial	Multiple	Partial	Multiple
U.S. Bonds:				
Treasury Bill	-1.903	2.149	-3.629	0.423
9-12 months	-3.174	1.751	-3.946	1.138
3-5 years	-2.116	1.071	-3.509	0.318
10+ years	-2.747	0.146	-2.970	-0.243
Corporate	-3.029	-1.511	-2.589	-1.207
Moody's	-1.846	1.992	-0.620	2.891
Canadian Bonds:				
Treasury Bill	-3.106	-1.323	-3.667	-1.384
1-3 years	-3.151	-3.124	-2.124	-2.106
3-5 years	-3.47	-2.815	-2.867	-2.451
5-10 years	-3.679	-1.49	-3.018	-1.006
10+ years	-2.818	-2.439	-1.951	-1.696
MeLeod, Young, Weir	-1.817	-2.528	-1.502	-2.358
	Domestic QCPI conditionalized on and in combination with domestic RPCG and RPCM.		Domestic YCPI conditionalized on and in combination with domestic RPCG and RPCM.	

FIGURE 6
Partial Coherence of MCPI Conditionalized on RPCM and RPCG With

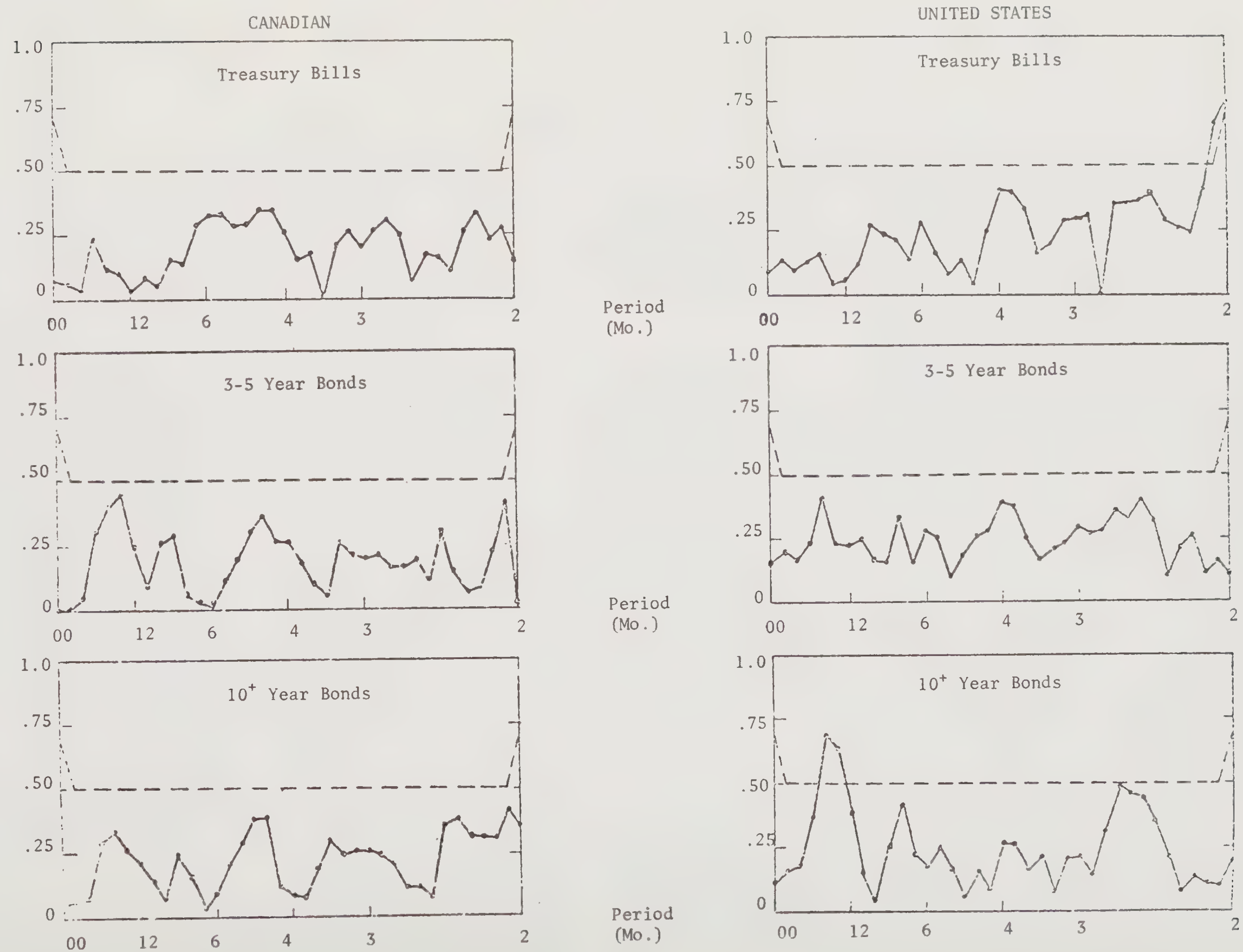
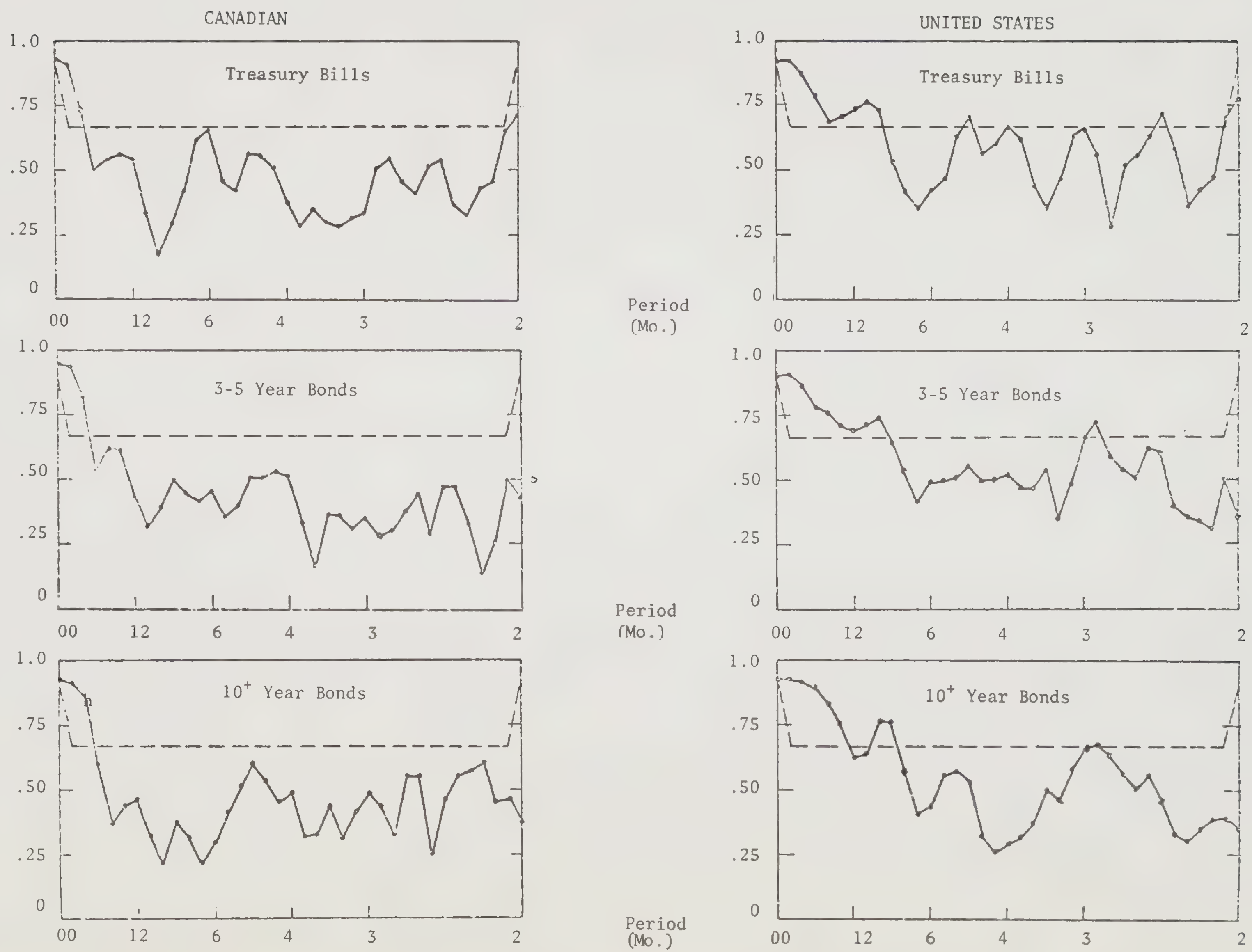


FIGURE 7
Multiple Coherence Between RPCG, RPCM and MCPI and



statistics and graphs of the partial coherence functions support roughly the same conclusions.[17] In fact, YCPI, the measure of inflation which previously performed best in terms of estimated regression coefficients, comes through with the weakest (partial) association in this multivariate context. The marginal coherence functions of interest rate series with YCPI characteristically have large and significant values in the low frequencies. (For example, see Figure 3.) However, the bivariate summary statistics reflect the overall lack of cohesion. The reason we tried alternative measures of inflation was to eliminate some of the erratic behavior present in the MCPI series. One of the effects of the smoothing procedures in the frequency domain is to increase markedly the proportion of the total variance of the smoothed measures of inflation attributable to low frequency components. The association in the low frequencies and the altered spectral shape of the smoothed measure of inflation act in concert to cause a dramatic change in an estimated function central to the Hannan regression routine - the signal-to-noise ratio. A representative signal-to-noise ratio for bivariate relationships involving MCPI tends to increase as frequency increases (Figure 5). A relatively flat signal-to-noise ratio is obtained when YCPI is used as the independent variable[18] (Figure 11). The result of this alteration in the signal-to-noise ratio is that any relationship which exists in the low frequencies is given more weight in determining the coefficients estimated with the Hannan routine.[19] This is the reason why the estimated coefficients are more significant when YCPI is the independent variable, despite the lower bivariate summary test statistics <u>vis à vis</u> MCPI and QCPI.

We have chosen not to consider the bivariate regression

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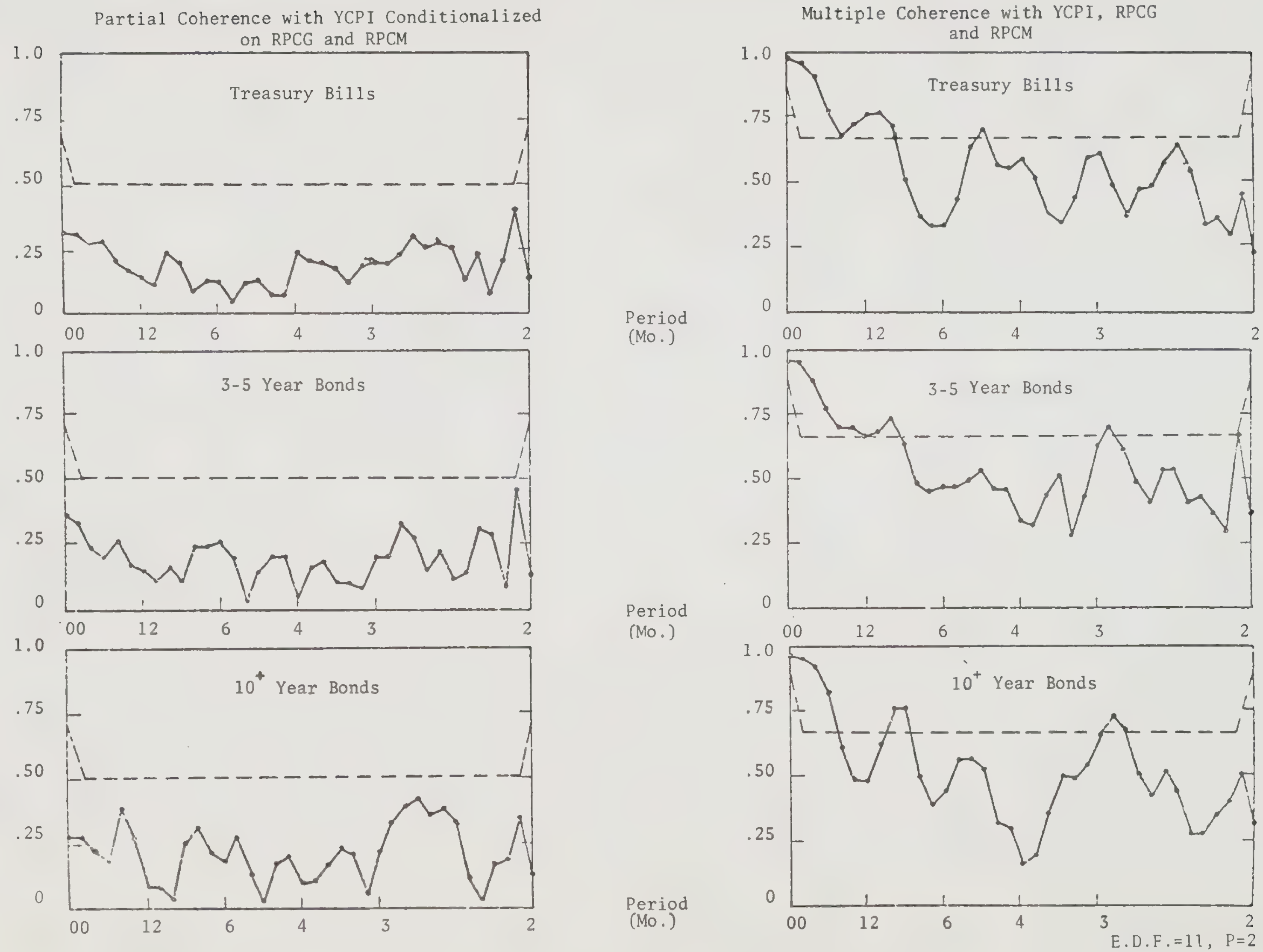
[17] The results using U.S. QCPI are quite similar to those obtained when U.S. MCPI was the third independent variable. A detailed discussion is, therefore, unnecessary.

[18] There are two ways to alter the signal-to-noise ratio - change the value of the residual spectrum or change the value of the spectrum of the independent variable. In this case, most of the alteration in the signal-to-noise ratio was due to changes of the second sort.

[19] See the discussion in Appendix C.

FIGURE 8
Coherence Functions For U.S. Interest Rates



results using YCPI as the independent variable as evidence supporting the Fisherian hypothesis. A reason for this is indicated in Figure 8, which contains both the partial and multiple coherence functions for selected interest rate series and YCPI, RPCG and RPCM. The partial coherence functions generally have low values in the low frequencies without an increase in the strength of the relationship in higher frequencies. In addition, the multiple coherence function of YCPI with RPCG and RPCM has large values in the low frequencies. This observation leads us to the conclusion that the generally significant coefficients obtained in our bivariate application of Hannan's estimation program reflect spurious marginal associations in low frequency components due to inadequate specification in the Naive Fisherian model.

Although it is possible to extend Hannan's method to estimate efficiently the parameters in the present single equation model or in multi-equation models (9), the computations are complicated and the problem of model specification (especially _a priori_ restrictions on the lag structure) are still present. On the other hand, the residual spectrum, estimated in a way which is analogous to that outlined in Appendix C, behaves erratically, prohibiting efficient estimation by less sophisticated techniques. For these reasons, no attempt was made to analyze these classes of multivariate models in the time domain.

Two major conclusions can be drawn from the analysis in this section. First, U.S. interest rate series generally cohere significantly with the classes of independent variables we considered. On the other hand, Canadian interest rates, when analyzed in the frequency domain, cannot be represented adequately by the same classes of variables. Second, no measure of the rate of inflation which we considered possesses any significant amount of independent explanatory power when included in multivariate models. This conclusion corroborates the conclusion of the previous section as we can find no support for the contention that linear combinations of the actual rate of inflation have significant associations with interest rates in the frequency domain. The uniformly low values of the partial coherence functions at virtually every frequency lead us to conclude that no significant time-domain association would emerge in a properly estimated multivariate model.

THE GIBSON PARADOX

The association between the level of domestic prices and domestic interest rates, labelled the Gibson Paradox by Keynes, has been analyzed historically with the bivariate correlation coefficient. We will look at this relationship in the frequency domain both in the customary bivariate context and in a multivariate context.

The bivariate and partial coherence function between the domestic price level and several bond rates are displayed in Figures 9 and 10 respectively. The bivariate coherence functions share one common characteristic: extremely large values in the very low frequencies with periods corresponding to two or more years. A peak in the coherence function also occurs at the frequency corresponding to a four-month cycle probably representing similarities in seasonal patterns of these series.

The corresponding partial coherence functions have considerably different shapes. They characteristically have relatively low values in the low frequencies with a peak in the coherence function in the vicinity of the frequency which corresponds to a one year period - again probably reflecting similar seasonal patterns. Comparisons of these two sets of figures support the general proposition that the confluence of two time series with general economic activity has been singled out for special attention. Indeed, this is one of the explanations Keynes [23] provided for the paradox.

Table VI contains the summary test statistics for the Gibson Paradox. As in the previous section, results for the Canadian series are quite weak. Also, the summary statistics and the non-parametric test tabled in the appendix support the conclusions obtained from these figures. No significant relationship exists between the price level and interest rates outside the relationship both have with RPCM and RPCG, with the possible exception of the Moody's series. In addition, the low frequency values of the multiple coherence function for the level of the U.S. CPI with the U.S. RPCG and RPCM are quite large, and the function has a summary test statistic value of 3.33. Although Keynes found the confluence explanation somewhat unpleasing, it appears to be a perfectly adequate explanation of the phenomena.

We note in passing that the level of domestic CPI has an

FIGURE 9
Gibson Paradox, Bivariate Coherences of The Level of Domestic CPI With

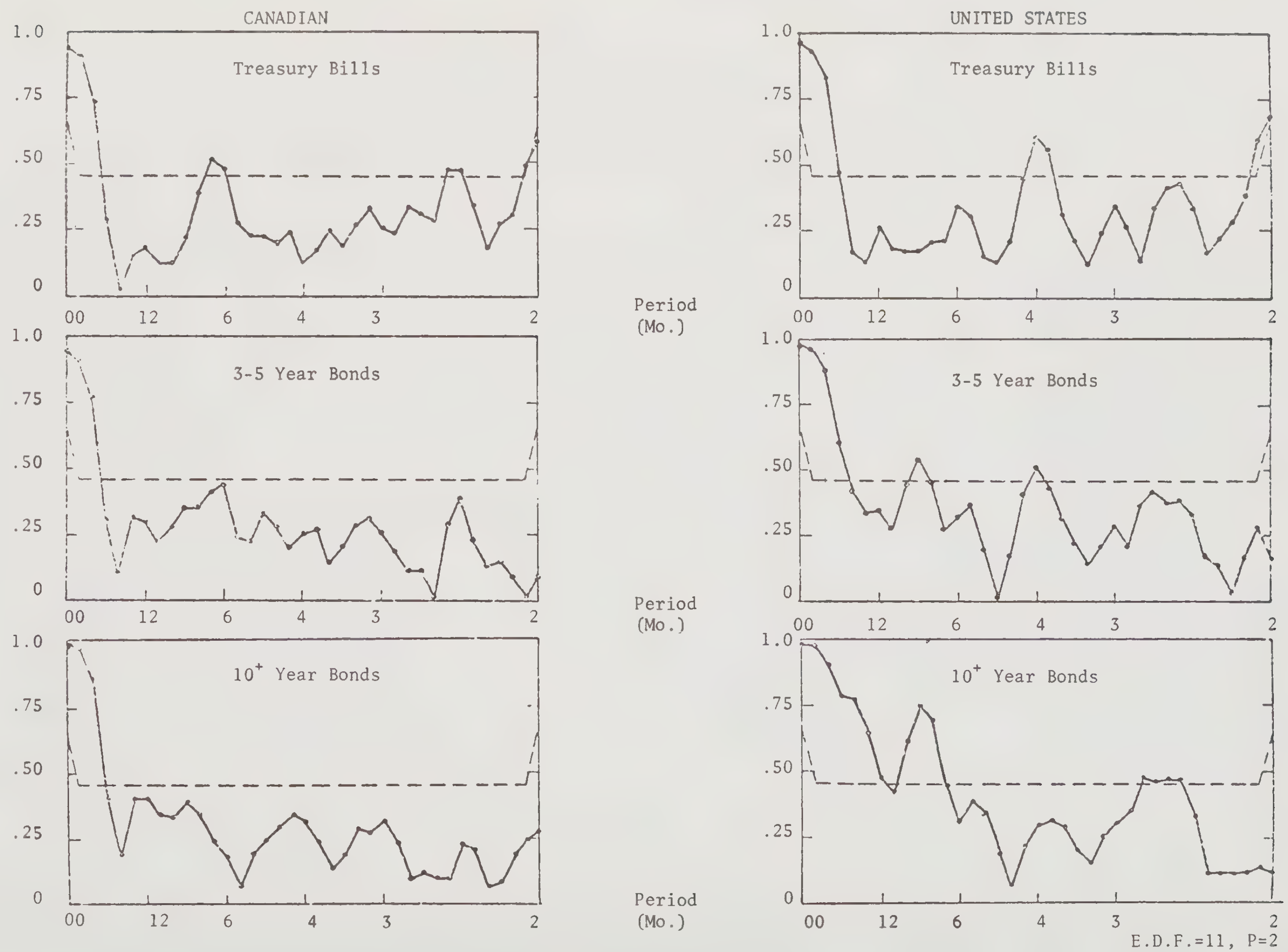


FIGURE 10
Partial Coherence Function With Level Domestic CPI
Conditionalized on RPCG and RPCM

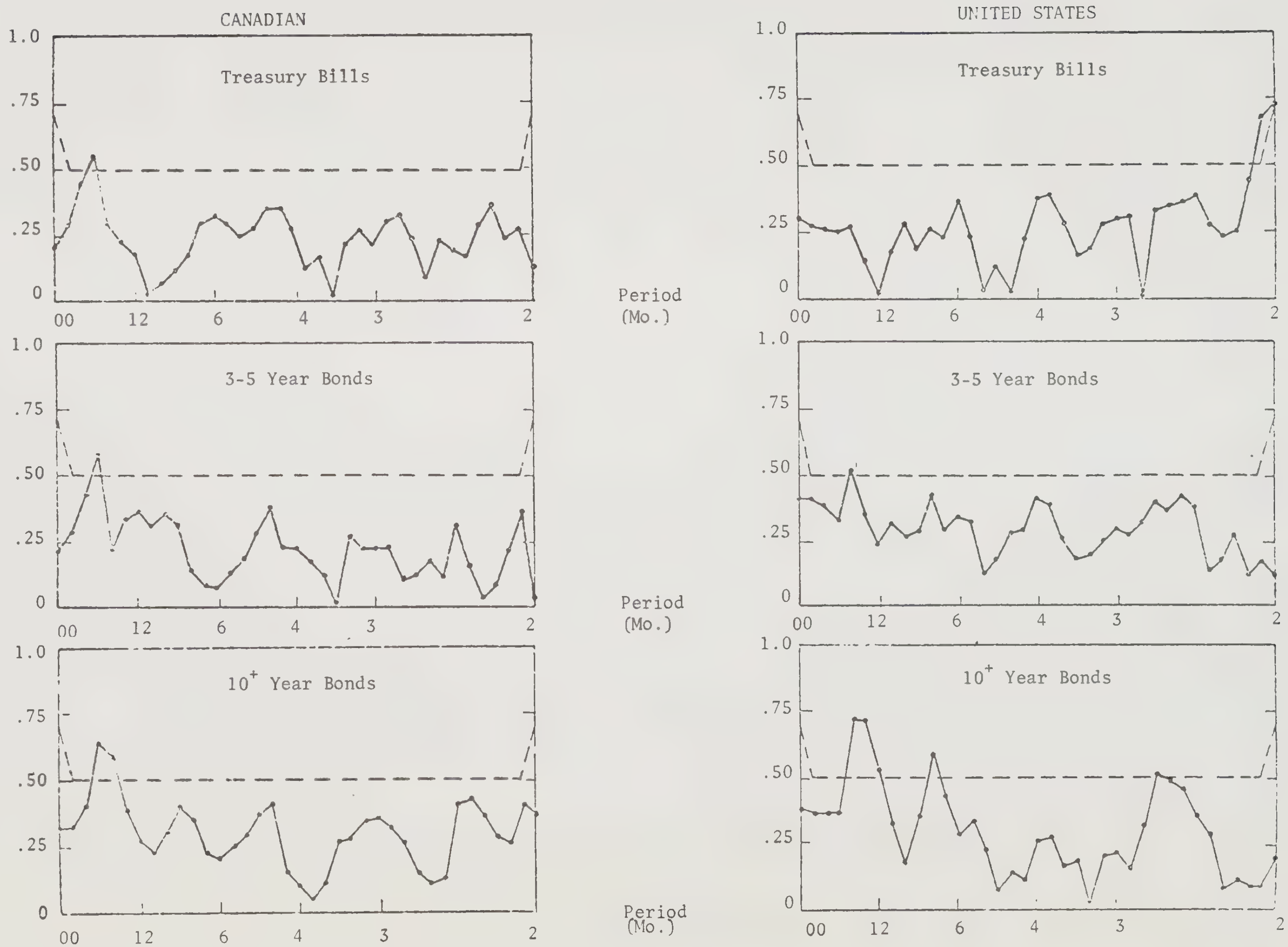
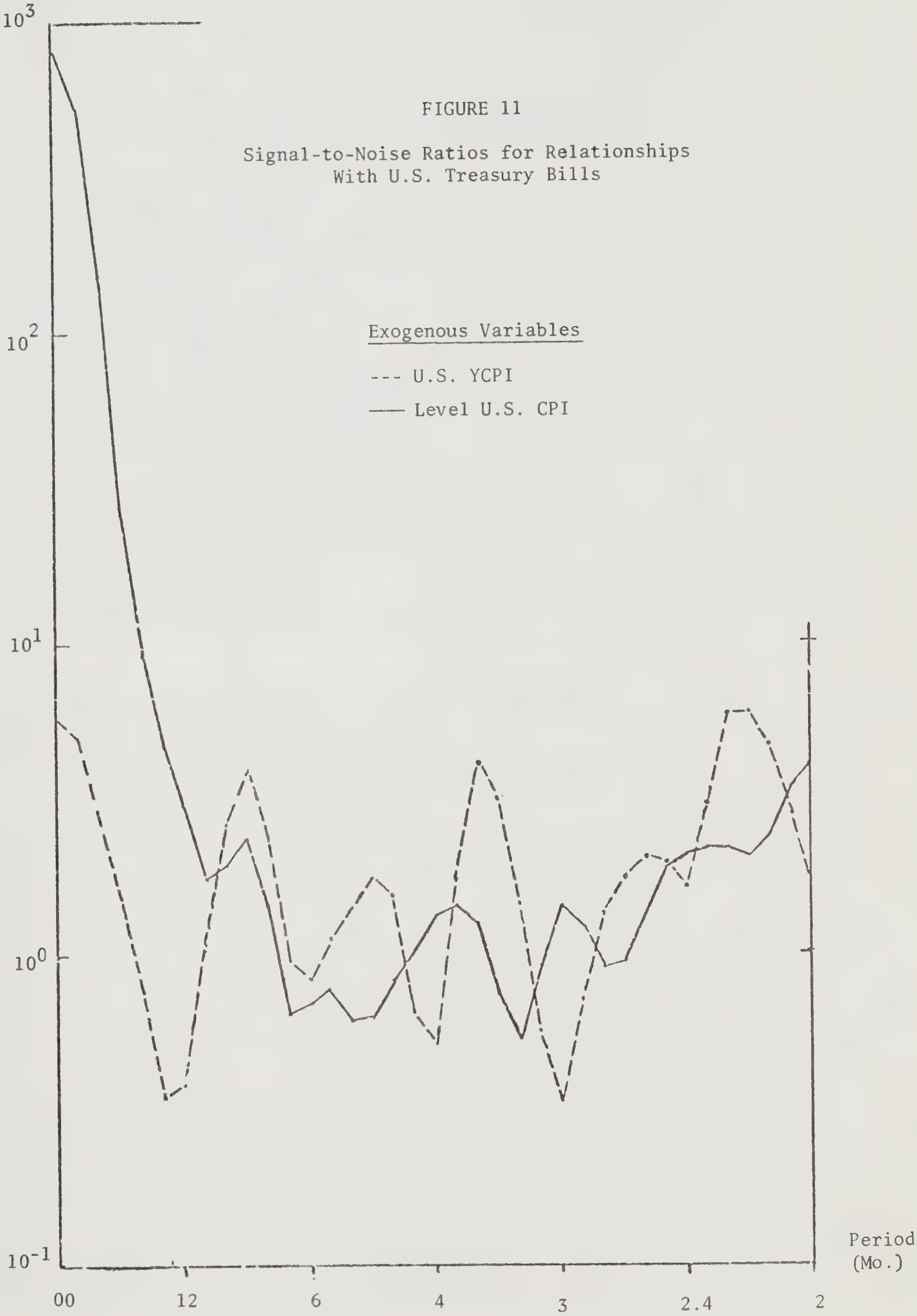


TABLE VI

The Gibson Paradox: Summary Statistics

	Marginal Coherence (with Level Domestic CPI).	Partial Coherence (Level Domestic CPI Conditionalized on RPCG and RCPM).	Multiple Coherence (with Level Domestic CPI, RPCG, and RPCM).
United States:			
Treasury Bills	2.311	-1.242	2.615
9-12 months	2.653	-1.796	2.513
3-5 years	2.802	-0.506	1.875
10+ years	4.051	-0.332	1.656
Corporate	2.924	-0.994	-0.167
Moody's	5.098	1.476	4.071
Canadian:			
Treasury Bills	1.446	-2.313	-1.002
1-3 years	0.016	-3.184	-3.094
3-5 years	0.408	-2.624	-2.484
5-10 years	1.376	-1.886	-0.593
10+ years	2.661	-0.042	-0.585
McLeod, Young, Weir.	2.499	0.001	-1.752



10^3
10^2
10^1
10^0
10^{-1}
FIGURE 11
Signal-to-Noise Ratios for Relationships
With U.S. Treasury Bills
Exogenous Variables
--- U.S. YCPI
—— Level U.S. CPI
00
12
6
4
3
2.4
2
Period
(Mo.)

even larger proportion of its total variance attributable to low frequency components than does domestic YCPI. There is a dramatic alteration in the signal-to-noise ratio and if regression coefficients were estimated with the Hannan routine even heavier weights would be applied to low frequency associations in the determination of the estimated coefficients (Figure 11).

CANADIAN AND UNITED STATES INTEREST RATES: TERM STRUCTURES AND INTERRELATIONS

If we assumed that capital-market participants had perfect foresight and that their transactions were costless, we would anticipate that the rate of return, including capital gains and gains due to changes in exchange rates, on bonds of any maturity from any country would be equal. The equilibrium rate of return resulted would be the rate of return determined by the international demand for and supply of capital. Complete equalization of the rates of return both across and within separate capital markets does not occur because conditions in the real world do not meet these assumptions. This section of the paper analyzes the equalization mechanism across and within the U.S. and Canadian capital markets. This study is descriptive; no formal models are developed although some relevant hypotheses are tested.

Once we introduce uncertainty about future interest rates, exchange rates, rates of inflation and solvency of issuers, we are faced with the nebulous and tautological proposition that the *a priori* rates of return on all bonds - in terms of utility - are equal. We can try to control some types of uncertainty and adjust the interest rates series (the observed quantity we are forced to use as a proxy for the rates of return) for other risks. By restricting our attention to government bonds and other high quality bonds we try to eliminate uncertainty about the solvency of the issuers. When comparing Canadian and U.S. rates, we adjust the U.S. rates for expected changes in the exchange rate. Of course, different rates of inflation provide the major long-run reason for exchange-rate adjustment and it would seem natural to dispose of both the internal and international uncertainty about real rates of return by taking account of this factor. However, the results of earlier sections indicate that there is no linear model which can be relied upon to adjust nominal interest rates for anticipated rates of inflation. Thus,

we will restrict the analysis to nominal interest rates on bonds which are free from the risk of issuer insolvency and are adjusted for expected exchange rate movements.

International interest rate linkage and linkage of rates on bonds of different maturities within a given country have been studied in depth, both theoretically and empirically. Our analysis employs some of the spectral techniques used by Granger and Rees [16] and extends the treatment to inter-country rates of interest. We appraise the frequency domain association between interest rate series for bonds of different maturities inside the United States and inside Canada, and for bonds of similar maturities across these two markets. The results of the spectral analysis are translated back into the time domain in order to facilitate understanding of the equalization mechanism and evaluation of period of adjustment. Naturally, both the U.S. and Canadian rates adjust to each other, but the amount of adjustment varies inversely with the size of the capital market. For convenience, we assume that if any equalization occurs, interest rates in the smaller Canadian market adjust to the rates on comparable bonds in the United States capital market. As in previous sections, estimates are based on 37 autocovariances and lagged cross-covariances computed from 223 monthly observations in the period from January 1952 to July 1970.

Since it is quite difficult to interpret phase functions, the timing of relationships is studied only in the time domain.[20] The focus of the frequency domain analysis will be on the strength of the relationship between the interest rate series. Other researchers have chosen to condense the information from the bivariate coherence functions by dividing the frequency domain into segments which correspond to cyclical movements economists customarily refer to as long, medium and short-run components (see [16]). This approach has one primary advantage: it allows us to see how the cohesion of the series varies over different cyclical components without inundating us with information. The drawbacks are that we still have three numbers and, more important, that divisions into different components are arbitrary. We get around these problems by using the assumed joint normality of the series to form the summary test statistic used earlier.

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[20] See Hause, [21].

TABLE VIII

Summary Statistics: U.S. Term Structure

	U.S. TB	U.S 9-12 mo.	U.S. 3-5	U.S. 10+	U.S. Corp.
U.S. TB	---	9.242 (0.984)	11.344 (0.962)	7.618 (0.893)	3.886 (0.869)
U.S. 9-12 mo.		---	16.0885 (0.978)	12.411 (0.925)	6.324 (0.878)
U.S. 3-5			---	16.615 (0.956)	7.995 (0.939)
U.S. 10+				---	9.002 (0.953)
U.S. Corp.					---

and United States bonds with similar characteristics: treasury bills, 3-5 year bonds, 10+ year bonds, and high grade corporate bonds. All the U.S. series were adjusted for variations in the exchange rate. We make the supposition that no opportunity for a covered arbitrage would go unexploited in a world characterized by zero transactions costs and competition between interest arbitragers. This supposition allows us to conclude that the difference between the domestic and foreign rates of interest equals the per annum premium (or discount) on domestic currency (See [29]). There will be approximate equality between adjusted intercountry interest rates when there are small, but non-zero transaction costs.

The adjustment is made by the use of the ninety-day forward rates of exchange and spot rates,[21] a procedure not entirely satisfactory for interest rate series other than the three-month treasury bill series. Unfortunately, the forward rate series one needs for adjustment of long-term bonds does not exist because currency transactions do not generally occur that far into the future. We feel that the correct direction of the adjustment is the same as that which is used for the treasury bill series. Hence, all U.S. series are adjusted by the same factor.

Table IX contains the summary test statistics and average coherences over the low frequencies for the relationship between the four Canadian and United States series. All the test statistics are significant at the one per cent level. The relatively low test statistic values for the 10+ and corporate bonds are a result of our inability to adjust properly for exchange rate risk. In fact, the unadjusted series for these bonds have considerably higher values of the test statistic, 9.379 and 6.693. Introduction of the correction factor for U.S. 10+ and corporate bonds has two effects. One is to decrease marginally the magnitude of the estimated cross spectra with their Canadian analogs. The other is to increase markedly the values of the spectra of the United States series in the high frequencies relative to their low frequency values. The two effects act in concert to diminish the values of the estimated coherence functions, especially in the high frequencies.

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[21] Spot and Forward rates of exchange for U.S. dollars in Canada were obtained from the research department of the Bank of Canada.

TABLE IX

Summary Statistics: U.S.-Canadian Interrelations

(Test Statistics with Average Coherences in Parentheses)

	U.S. TB Adj.	U.S.3-5 Adj.	U.S.10+ Adj.	U.S. Corp. Adj.
CAN. T.B.	5.43 (0.897)			
CAN. 3-5		5.42 (0.856)		
CAN. 10+			3.483 (0.690)	
McLeod Young Weir				2.973 (0.754)

The improvement in the summary test statistics for the relationship between the Canadian and the adjusted U.S. treasury bonds and 3-5 year bonds is not dramatic relative to values for the unadjusted series (5.83 and 5.17 respectively.) However, it is factuous to choose between the adjusted and the unadjusted U.S. series on the basis of the test statistic because the values are so high that we would be trying to discriminate between levels of type I error of the order of 0.001 versus 0.00001. Thus, we shall continue the analysis using the adjusted series with the caveat that we may well have inadvertently introduced an error in variables problem involving the U.S. 10+ and corporate bond rate series by improper adjustment for exchange risk.

Figure 12 presents the coherence functions for the intracountry comparisons. Roughly every other point on the estimated coherence function is independent, and those points above the dotted line are statistically significant at the 10 per cent level. The estimated low frequency coherences are generally significant.

We now turn to estimation of parameters for intercountry and intracountry relationships between interest rate series

in models of the form:

$$Y_t = \sum_{v=s}^{r} \beta_v X_{t-v} + \alpha \qquad (4);$$

where α is a constant, β_v are distributed-lag coefficients, r and s are unspecified integers, X_{t-v} is the value of the independent variable at time t - v, and Y_t is the value of the dependent variable at time t. It should be noted that these models allow for distributed leads as well as lags in cases where $s < 0$. Primarily statistical considerations lead us to include future values of the independent variable. This procedure can be justified on economic grounds if we make several assumptions about how market participants decide what constitutes a reasonable nominal rate of return on a given bond relative to the nominal rate of return on key alternative types of investment. First, investors may require a liquidity premium on relatively long-term bonds which may be invariant with respect to the general level of interest rates. Such a premium would be picked up by the constant term. Second, both sides of the market base their judgments about a reasonable nominal rate of return on a given bond (denoted by Y) partially on extrapolative and partially on predictive analysis of the rate of return on the key alternative investments (denoted as X). In other words, while the participants in the market recognize the strong long-term trend factors dominating interest rate movements for all bonds, they also recognize that non-quantifiable, though assessable factors affect the future course of X which, in turn, affects the future course of Y. The market participants may then arrive at a market interest rate for a given bond Y which reflects past rates as well as predicted rates of X. In this analysis we use actual future rates as proxies for the predicted rates. Further, we also assume that the rate of interest which the participants arrive at in the marketplace is a weighted sum of past and future values of X.

The summary test statistics in the previous section indicate that there are many strong relationships between intercountry and intracountry interest rate series from which to choose. We explore only a small subset of these models. One group of models we analyze embodies the proposition that major movements in the term structure of interest rates originate with movements in the rates of interest on the most

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Figure 12 presents the coherence functions for the intracountry comparisons. Roughly every other point on the estimated coherence function is independent, and those points above the dotted line are statistically significant at the 10 per cent level. The estimated low frequency coherences are generally significant.

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in models of the form:

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where α is a constant, β_v are distributed-lag coefficients, r and s are unspecified integers, X_{t-v} is the value of the independent variable at time t - v, and Y_t is the value of the dependent variable at time t. It should be noted that these models allow for distributed leads as well as lags in cases where $s < 0$. Primarily statistical considerations lead us to include future values of the independent variable. This procedure can be justified on economic grounds if we make several assumptions about how market participants decide what constitutes a reasonable nominal rate of return on a given bond relative to the nominal rate of return on key alternative types of investment. First, investors may require a liquidity premium on relatively long-term bonds which may be invariant with respect to the general level of interest rates. Such a premium would be picked up by the constant term. Second, both sides of the market base their judgments about a reasonable nominal rate of return on a given bond (denoted by Y) partially on extrapolative and partially on predictive analysis of the rate of return on the key alternative investments (denoted as X). In other words, while the participants in the market recognize the strong long-term trend factors dominating interest rate movements for all bonds, they also recognize that non-quantifiable, though assessable factors affect the future course of X which, in turn, affects the future course of Y. The market participants may then arrive at a market interest rate for a given bond Y which reflects past rates as well as predicted rates of X. In this analysis we use actual future rates as proxies for the predicted rates. Further, we also assume that the rate of interest which the participants arrive at in the marketplace is a weighted sum of past and future values of X.

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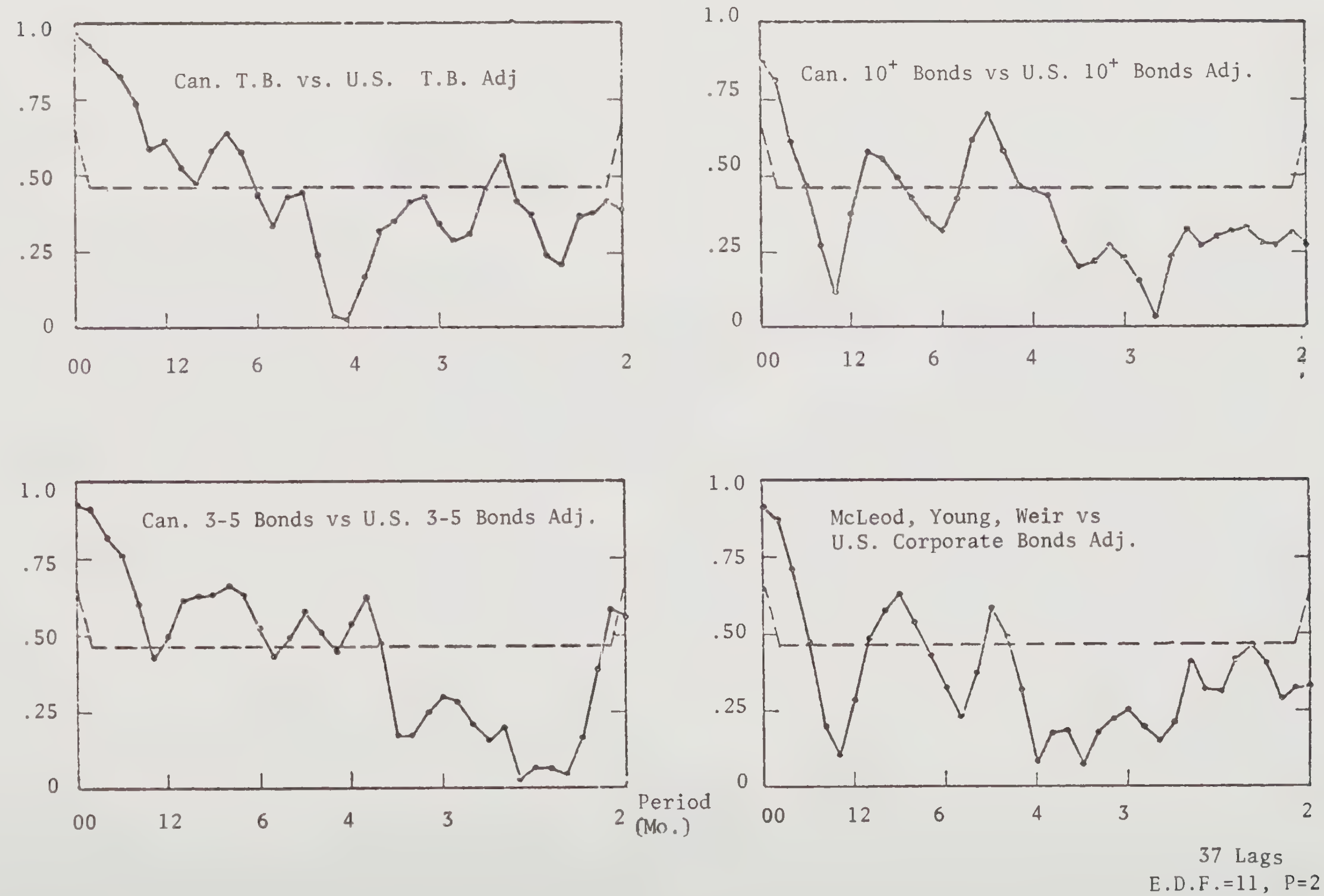
FIGURE 12

Selected Marginal Coherence Functions



liquid type of bond. Hence, we estimate models taking the domestic treasury bill rate as the independent variable, X, and other domestic government and high quality bond rates as the dependent variables, Y. The other group of models encompasses the proposition that Canadian interest rates adjust to U.S. interest rates. We can approach this group in either of two ways. We could suppose that the rates adjust ultimately to the adjusted U.S. Treasury Bill rate, or we could suppose that they adjust to rates on bonds with comparable characteristics. The first supposition is a composite of the proposition underlying the first group of models and the supposed intercountry ties. The data could support either premise, but we present results for models using the adjusted rates on bonds with similar characteristics because they rest on somewhat more direct and simpler hypotheses of behavior.

We again use the generalized regression technique suggested by Hannan [18,20], which relies heavily on the sample residual spectrum and signal-to-noise ratios. A representative example of each of these functions for the models we are now considering is presented in Figure 13. Time domain models with independent and identically distributed residuals or residuals which follow a first order autoregressive scheme correspond to residual spectrums which are flat or monotonic, respectively. The estimated residual spectrums for the models we consider were neither flat nor monotonic, indicating that the residuals follow no simple pattern. As in section two, estimation of parameters by the usual techniques would be inefficient.

The results of our generalized least squares analysis are presented in Table X. The estimated coefficients, b_V, along with their test statistics in parentheses, are displayed in labelled columns. The test statistic is the test of the null hypothesis that the specific regression coefficient is less than or equal to zero, and the statistic should be compared to the standard normal distribution. The constant of the regression, a_V, is also presented. The criterion mentioned in Appendix C is included for each regression.[22] The per

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[22]The criterion is the summary test statistic for testing whether the coherence between the exogenous variable and the residual proces is zero. The estimated model which (cont'd)

cent of variance explained is the ratio of the explained to the total variance of the dependent variable. It can be used as a rough measure of the independent variable's capacity to account for variation in the dependent variables. The last rows in the table are the sum of the regression coefficients and a test of the hypothesis that the coefficients sum to unity. The asymptotic distribution of this test statistic is also the standard normal distribution under the null hypothesis, and hence, the critical region at the five per cent level is the region (X:x > 1.96 or x < -1.96).

Some general observations about these results are in order. First, the estimated regression coefficients meet our a priori expectation in that they are usually positive and less than one. The largest and most significant coefficient is that associated with the contemporaneous value of the independent variable. Also, the value of the contemporaneous regression coefficient decreases as the lengths of maturity of the dependent interest rate series increases. The fit of the equations is reasonably good, and the values of the criteria indicate that we can accept the hypothesis that the residual and the exogenous variable are independent at the 10 per cent level in all but one case.

The within-country equations possess many of the properties we ascribe to them on an a priori basis. The estimated constant generally increases as the length of maturity increases. Since we use the treasury bill interest rate series as the independent variable, we expect the strength of the time domain association, measured by the per cent of variance explained, to decrease as we move to longer-term bonds. This expectation is realized with one exception. It might also

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22 (Continued)
is tabled is the model which minimized this criteria from the class of parsimonious models. The word "parsimonious" is used to indicate that we did some preliminary screening of the models. The rough criterion for this screening was that the model should have a high proportion of its coefficients (negative or positive) significantly different from zero. Every estimated model could have been tested, but computer costs would have been unacceptably large, and we would still have no assurance that the global minimum of the criterion had been found.

FIGURE 13

Residual Spectrum and Signal-to-Noise Ratio for
Canadian T.B. as a Linear Function of U.S. T.B., Adjusted

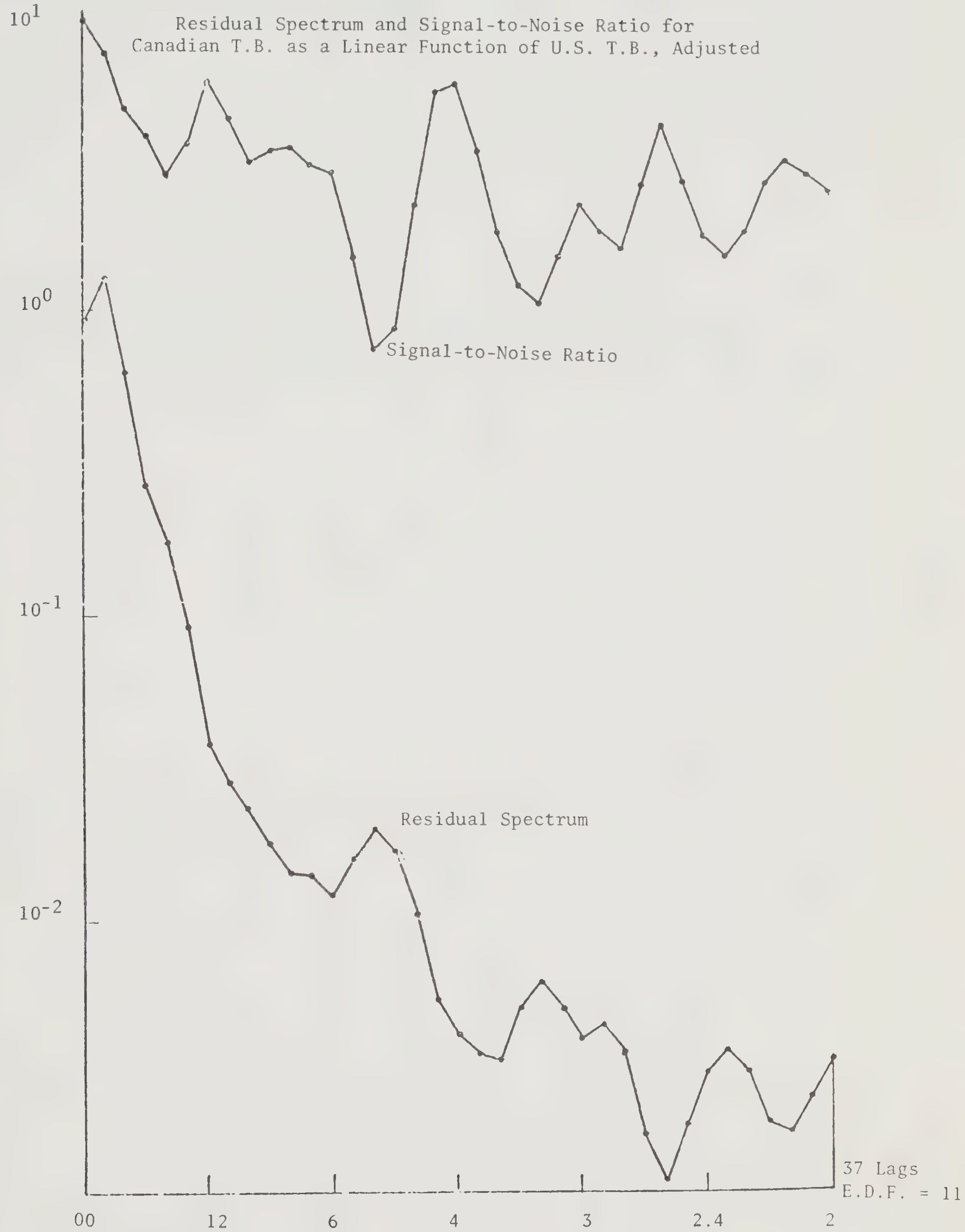

TABLE X

Y_t	CAN. T.B.	CAN. 3-5	CAN. 10+	McLeod, Young, Weir.
X_{t-v}	U.S. T.B. Adj.	U.S. 3-5 Adj.	U.S. 10+ Adj.	U.S. Corp. Adj.
b_{t-3}	0.085 (2.015)		0.058 (2.445)	0.041 (2.376)
b_{t-2}	0.075 (1.779)		0.06 (2.45)	0.112 (5.906)
b_{t-1}	0.196 (4.652)	0.154 (4.482)	0.089 (3.583)	0.127 (6.626)
b_t	0.267 (6.374)	0.268 (7.784)	0.18 (6.857)	0.148 (7.439)
b_{t-1}	0.190 (4.537)	0.154 (4.482)	0.098 (3.7)	0.136 (6.827)
b_{t-2}	0.081 (1.924)		0.079 (2.923)	0.126 (6.304)
b_{t-3}	0.088 (2.085)		0.083 (3.023)	0.057 (2.982)
b_{t-4}	-0.016 (-0.389)		0.069 (2.552)	0.049 (2.572)

b_{t-5}	0.05 (1.197)		0.061 (2.309)	0.043 (2.415)
b_{t-6}			0.033 (1.257)	
b_{t-7}			0.025 (1.002)	
b_{t-8}			0.027 (1.092)	
b_{t-9}			0.03 (1.269)	
a (Const.)	-0.238	2.058	0.908	1.415
Criterion	1.621	0.465	0.449	0.675
% of Var. Explained	0.834	0.687	0.61	0.746
Sum of Reg. Coeff.	1.016	0.574	0.867	0.839
Test of Complete Adjustment	0.16	-7.35	-0.74	-1.63

TABLE X (continued)

Y_t	CAN. 1-3	CAN. 3-5	CAN. 10+	McLeod, Young, Weir.
X_{t-v}	CAN. T.B.	CAN. T.B.	CAN. T.B.	CAN. T.B.
b_{t-3}				0.035 (1.887)
b_{t-2}			0.03 (1.344)	0.035 (1.905)
b_{t-1}	0.078 (2.467)	0.111 (3.61)	0.08 (3.516)	0.098 (5.199)
$\bar{b}_t$	0.649 (18.61)	0.489 (15.111)	0.208 (9.168)	0.192 (10.12)
b_{t-1}	0.069 (1.978)	0.111 (3.61)	0.082 (3.615)	0.105 (5.573)
b_{t-2}	0.039 (1.188)		0.028 (1.231)	0.039 (2.066)
b_{t-3}			0.056 (2.487)	0.042 (2.188)
b_{t-4}				0.044 (2.312)

b_{t-5}				0.029 (1.544)
b_{t-6}				0.028 (1.487)
b_{t-7}				0.033 (1.735)
b_{t-8}				0.042 (2.224)
b_{t-9}				0.017 (0.912)
a (Const.)	1.271	2.047	3.181	3.015
Criterion	-0.398	0.107	1.263	1.339
% of Var. Explained	0.947	0.927	0.773	0.895
Sum of Reg. Coeff.	0.835	0.711	0.484	0.74
Test of Complete Adjustment	-5.00	-3.82	-9.4	-3.08

TABLE X (continued)

Y_t	U.S. 9-12 mo.	U.S. 3-5	U.S. 10+
X_{t-v}	U.S. T.B.	U.S. T.B.	U.S. T.B.
b_{t-3}			
b_{t-2}			0.073 (2.619)
b_{t-1}	0.127 (3.723)	0.176 (4.829)	0.104 (3.738)
b_t	0.812 (21.928)	0.468 (12.042)	0.256 (9.084)
b_{t-1}	0.127 (3.723)	0.132 (3.31)	0.104 (3.738)
b_{t-2}		0.063 (1.581)	0.073 (2.619)
b_{t-3}		0.05 (1.245)	
b_{t-4}		0.021 (0.538)	

b_{t-5}		-0.005 (-0.145)	
b_{t-6}			
b_{t-7}			
b_{t-8}			
b_{t-9}			
a (Const.)	0.095	0.94	1.981
Criterion	1.057	1.663	0.572
% of Var. Explained	0.979	0.949	0.899
Sum of Reg. Coeff.	1.066	0.905	0.61
Test of Complete Adjustment	1.94	-2.02	-8.66

be suspected that the coefficients for the values of the exogenous variable would sum to unity. We accept this supposition at the five per cent level in only one of the seven cases. We reject this hypothesis for all of the Canadian equations.

Three of the four intercountry equations provide support for the posited complete equilibrium of Canadian rates to adjusted U.S. rates. It can be argued that the constant reflects how well we adjusted the U.S. series for the risks involved in investing in Canadian securities. That argument runs along the line that the adjusted U.S. series should have a mean quite close to the mean of the similar Canadian series. If the equilibrium of international interest rates were complete, the true value of the estimated constant would be near zero. The estimates of the constant are quite different from zero in three of the cases.

Although the percentage of explained variance is generally somewhat less than that obtained for intracountry distributed lag equations, we conclude from both our time and frequency domain analysis that movement of Canadian interest rates can be adequately explained by the movements of rates on comparable U.S. bonds.

CONCLUSION

We must first stress the limitations of this analysis - particularly of the first sections. The relationship between the expected rate of inflation and nominal interest rates is the central focus of the first sections of this study. We have had to be content with testing variants of the most rudimentary formulations of this hypothesis, deriving price expectations from linear combinations of actual rates of inflation and making other rather unpalatable assumptions about the determinants of the real rate of interest. The use of monthly data may be considered another limitation; however, this is not necessarily a disadvantage when something as potentially volatile as expectations or as capable of rapid adjustments as capital markets are studied. Indeed, annual or quarterly data would probably be too smooth to pick up the effects we have studied.[23] In addition, monthly data

[23] As Mundlak [25] points out, the period of adjustment is

do not necessarily hopelessly bury relationships in noise.

Within these limitations, we can draw several conclusions. First, using monthly rates of change of the CPI as the measure of inflation, we find no evidence of linear effects of actual inflation on nominal interest rates either in bivariate or in multivariate contexts. Second, we find no relationship when quarterly rates are taken as the measure of inflation. Finally, we find that YCPI and nominal interest rate series have no significant relationship. The results involving YCPI were the only ones upon which an argument for the supposed relationship could be made. However, what supporting evidence we found in the bivariate regression coefficients must be heavily discounted. As we move toward smoother measures of inflation, transformations of the spectrum of the measures are such that virtually unchanged interrelationships are able to produce significant regression coefficients. Because they rest on associations which are severely eroded by inclusion of other relevant variables, we conclude that the results using YCPI were the product of misspecification in the bivariate model. Indeed, the thread that ties together our analysis of the Fisherian hypothesis and the Gibson Paradox is just that observation: While smoother measures of a variable may allow a researcher to produce a significant relationship in the time domain, such relationships rest on low frequency associations whose source is often identifiable only by prejudice.

If the reader's prejudices are such that he finds support for the Naive Fisherian hypothesis in this analysis he still should draw an ancillary conclusion: the lags in response to inflation in financial markets are considerably shorter than commonly suspected. The bivariate regression results rarely indicate significant coefficients for the annual rates of change of the CPI lagged more than nine months.

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23 (cont'd)
critically important in distributed lag estimation and "if the adjustment is nearly complete in a period that is short relative to a year, annual analysis should not be dynamic". The long lags in response suggested by previous research comes, in part, from improper specification of the period of adjustment and improper aggregation over time. Also, see Zellner and Montmarquette [33].

An inference having more fundamental application may also be drawn. We could view this analysis as a conditional test of the hypothesis that expected inflation is constructed by linear combinations of actual inflation. It is conditional because we must assume that the Fisherian formulation of the determinants of nominal interest rates embodied in equation (1) is correct and that the other exogenous variables we have included in the analysis account satisfactorily for the real interest rate term. We could then infer that linear combinations of actual inflation do not adequately represent expected inflation. Those disposed to the ancillary conclusion would infer that expectations quickly adapt to changes in actual inflation - so quickly, in fact, that annual and quarterly data could not reasonably reflect the adjustment process.

In any event, the results we obtained are markedly at odds with the results of most other studies. Models which bear more than a passing resemblance to our rudimentary Fisherian models have been used to analyze the impact of inflation on the real interest rate or to develop an expected rate of inflation series to be fed into other parts of large econometric models. As a result of this study, we conclude that such models are deficient.

The results of our intracountry analysis generally agree with the results obtained in other studies.[24] First, the results of the frequency-domain analysis suggest an extremely strong association between rates on bonds with different maturities. We conclude that the usual term structure of interest rates applies. Our time-domain analysis verifies the existence of the strong association indicated by the frequency domain results. We also found that, in general, the adjustment of long-term rates to movements in the domestic

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[24] Other researchers have recently been using the same techniques for analyzing the term structure of interest rates. Cargill and Meyer [3] use Hannan's estimator and obtain similar results although they treat the data differently and work with slightly different time series. These authors specify the lag structure a priori rather than using a criteria to select the distributed lag model. Sargent [28] uses the inefficient version of Hannan's estimator, assuming that the signal-to-noise ratio is constant.

treasury bill rate was not complete. The only intracountry relationship which indicated complete adjustment was that between the series on U.S. 9-12 month bonds and U.S. Treasury Bills. These are the two series closest together in terms of length of maturity. The lack of complete adjustment, especially over periods in which trends in interest rates are generally upward, has been noted by other analysts [22]. On the other hand, our results are at odds with many other studies in that we found much shorter lags in adjustment (1 - r - s months) than usually obtained. For example, one model had a distributed lag covering 13 months while the rest covered seven or fewer months.

The intercountry association, both in the time and frequency domains, proved to be quite strong. Thus, the results of our intercountry analysis support the proposition that Canadian interest rates are, by and large, determined by the adjusted rates on similar U.S. bonds. Equalization was found to be complete in three cases, and in no case did the adjustment period cover more than 13 months.

From the point of view of Canadian monetary authorities, this analysis has two implications. First, because Canadian interest rates move closely with comparable U.S. rates, the monetary authorities' ability or willingness to bring forth independent movements in Canadian rates to implement domestic economic policy is seriously in question. In the present analysis we are unable to distinguish whether the Canadian monetary authorities have voluntarily matched movements in U.S. interest rates to avoid erratic capital flows or whether market forces have brought about the close association between the series independent of the actions of the Canadian monetary authorities. We are disposed to the latter explanation, although there is no doubt an element of truth in both. If the confluence of the interest rate series is largely the product of market forces, any efforts by the Canadian authorities to alter independently these interest rates is hindered by the fact that the period for complete intercountry equilization is short and the estimated regression coefficients around the contemporaneous value of the rate on the similar U.S. bonds are relatively large. Whatever the cause of the association, economic analysts should not rely on Canadian interest rates as indicators of monetary ease or tightness. Second, we found that no linear relationships existed between rates of inflation and interest rates. Hence, mechanical reliance on simple linear models for adjustment

of nominal interest rates for determining real rates of interest is not warranted.

APPENDIX A

DATA

We have used data from a variety of sources. Table A.1. contains the most important information concerning the origins and constructions of the variables.

There are serious problems involved in testing these time series for normality. First, we must cope with the question of whether we test the raw data or the prewhitened data. Prewhitening improves a series approximation to normality, and we can reject some of the raw data's conformity to normality by inspection (non-constant mean values). In addition, we cannot establish the hypothesized multivariate normality of the time series by accepting the marginal normality of the individual series. In short, we are in the position of trying to appraise the validity of an untestable proposition. We give the results of the Mann-Wald Chi Square test of the marginal normality of each prewhitened time series in Table A.2. They serve as guides and, in some cases, warning signals that tests of hypotheses which involve series with high chi square statistics should be viewed with a jaundiced eye.

It is possible to choose a linear filter for each series which will yield a prewhitened time series compatible with the marginal normality assumption. However, the various co-

herence functions are of central interest. These functions are, in the limit, invariant with respect to linear transformations of the raw data (see Appendix B). The main reason for prewhitening the time series is to reduce the number of lags required to obtain reasonable resolution of the estimated spectra. In general, a large percentage of the variance of economic time series is located in the low frequencies. Since all spectral windows have some leakage (nonzero bandwidths), the values of the spectrum at nearby frequencies affect estimates at any given frequency. It is therefore advantageous to decrease disparities in the amount of spectral mass located in adjacent frequency. This operation is generally accomplished for economic time series by choosing a filter which attenuates the low frequency components of the spectrum and amplifies the higher frequency components. The filter which we chose was $(1 - 0.70L)^3$, where L stands for the lag operator, $LX_t = X_{t-1}$. (The problem with using several filters is that the recoloring phase is complicated and it is laborious to write a program sufficiently flexible to handle the series properly under all hypotheses of interest.) To illustrate the filter's effect, a portion of the prewhitened and raw interest rate series on U.S. Treasury Bills is charted on Figure A.1. A trend is present in the raw rates while the prewhitened rate moves randomly about its mean.

Table A.3 contains results of a test of the stationarity of the individual time series used in this analysis. The test is performed by dividing each time series into two periods of equal length and then estimating the spectral density function for each subperiod (denoted by $g_{xx}'(\lambda)$ and $g_{xx}''(\lambda)$). We then chose L - 1 roughly independent estimates of each spectral density function. After stabilizing the variance of our estimates by taking natural logs of those estimates, we form the statistic

$$D = (L + 1)^{-1} \sum_{i=0}^{L} \quad [\ln g_{xx}'(\lambda) - \ln g_{xx}''(\lambda_i)] / \sqrt{.2454\delta_i}\ ,$$

$$\text{where} \quad \delta_i = \begin{cases} 2, \ i = 1,L \\ 1, \ \text{otherwise} \end{cases}$$

$$\text{and} \quad \lambda_i = (i - 1)\ \pi/L.$$

We will test the hypothesis that the X_t series is stationary by comparing the D value entered in Table A.3 to the standard normal distribution. Note that we are only testing for the stationarity of the individual series, not their joint stationarity.

TABLE A.1

SOURCES OF DATA AND CONSTRUCTION OF VARIABLES

(All Data Used Were Raw Data, Not Seasonally Adjusted)

TYPE	SOURCE	Method of Collection	Mode of Construction
Canadian Treasury Bills	Bank of Canada Research Department	Thursday Tender Following Last Wednesday of the Month.	--------------------
Canadian 1-3 year, 3-5 year, 5-10 year, and 10+ Bonds.	Bank of Canada Research Department	Average of Month Ends Before 1959; Subsequently, Average of Wednesdays.	--------------------
McLeod, Young, Weir Rates	McLeod, Young, Weir & Company Limited	Average of Ten Industrials at Month's End.	--------------------
Canadian Consumer Price Index (CPI) Level	DBS 62-002. Prices and Price Indexes.	Price Index, 1961=100 (All Items), Monthly	--------------------
Canadian MCPI	Same as Above	Same as Above	$100x[(CPI_t/CPI_{t-1})^{12}-1]$

TYPE	SOURCE	Method of Collection	Mode of Construction
Canadian QCPI	Same as Above	Same as Above	$100x[(CPI_t/CPI_{t-3})^4-1]$
Canadian YCPI	Same as Above	Same as Above	$100x[(CPI_t/CPI_{t-12})-1]$
Canadian M_2	Bank of Canada Statistical Summary	Average Wednesdays for Month.	Currency Outside Banks plus Demand Deposits (less Canadian Dollar Float) plus Personal Savings Deposits held in Banks.
Canadian Population (Total)	DBS 91-201, Estimate of Population of Canada by Province.	Available Quarterly.	Linear Interpolation to obtain Monthly Estimates.
Canadian Gross National Product (GNP)	DBS 13-531, National Income and Expenditure Accounts.	Available Quarterly.	--------------------
Total Labor Income, Canada	DBS 72-005, Estimates of Labor Income (Monthly and Quarterly).	Available Monthly and Quarterly.	--------------------
Canadian Real Per Capita GNP (RPCG)	Constructed	----------------	$RPCG = 100/CPI \times \frac{\text{GNP (Quarterly)}}{\text{Labor Income Quarterly}} \times$

TABLE A.1 (continued)

TYPE	SOURCE	Method of Collection	Mode of Construction
Canadian Real Per Capita GNP (RPCG) Cont'd.			$\frac{\text{Labor Income (Monthly)}}{\text{Population (Monthly)}}$.
Canadian Real Per Capita M_2 (RPCM)	Constructed	------------------	$\frac{100}{\text{CPI}} \cdot \frac{M_2 \text{ (Monthly)}}{\text{Population (Monthly)}}$
U.S. Treasury, 9-12 month and 3-5 yr. Bonds	Federal Reserve Bulletin	Average of Daily Closing Bid Prices	----------------
U.S. 10+ yr. Bonds	Same	Daily Average for Bonds Maturing or Callable in 10 or More Years.	----------------
Moody's	Same	Aaa Rated State and Local Government Bonds Based on Thurs-day Figures.	----------------
Corporate Bonds	Same	Industrial Bond Ser-ies, Average of Daily Figures.	----------------

U.S. Consumer Price Index, Level (CPI)	The Biennial Supplement to the Survey of Current Business.	1957-1959 = 100 (All items)	----------------
U.S. MCPI	Same as Above	Same as Above	$100\text{x}[(CPI_t/CPI_{t-1})^{12}-1]$
U.S. QCPI	Same as Above	Same as Above	$100\text{x}[(CPI_t/CPI_{t-3})^{4}-1]$
U.S. YCPI	Same as Above	Same as Above	$100\text{x}[(CPI_t/CPI_{t-12})-1]$
U.S. M_2	Federal Reserve Bulletin	Average of Daily Figures.	Currency Held by Public plus Demand Deposits plus Saving Deposits in Banks.
U.S. Gross National Product	The National Income & Product Accounts of the United States, 1929-1965, 1966-1970 Contained in July Issue of Survey of Current Business.	Quarterly	---------------
U.S. Population	U.S. Department of Commerce: Population Estimates and Projections.	Resident Population, Estimates Available Monthly.	---------------
U.S. Industrial Production	Survey of Current Business.	Total Index (Including Utilities), 1957-59=100 Monthly.	Quarterly Figures are Sums of Corresponding Monthly Figures.

TABLE A.1 (continued)

TYPE	SOURCE	Method of Collection	Mode of Construction
U.S. Real Per Capita GNP (RPCG).	Constructed	---------------	U.S. RPCG = 100/CPI x $\frac{\text{Industrial Production (monthly)}}{\text{Industrial Production (quarterly)}}$ x $\frac{\text{GNP (Quarterly)}}{\text{Population (Monthly)}}$.
U.S. Real Per Capita M_2	Constructed	---------------	$\frac{100}{\text{CPI}} \times \frac{M_2 \text{ (Monthly)}}{\text{Population (monthly)}}$

TABLE A.2

MANN-WALD CHI SQUARE TEST OF NORMALITY OF PREWHITENED DATA

	CANADIAN		U. S.	
Level of Test (Type I Error)	0.1	0.01	0.1	0.01
Treasury Bills	63.09	55.16	42.47	36.84
9-12 Month Bonds	-----	-----	41.17	30.47
1-3 Year Bonds	51.31	32.76	-----	-----
3-5 Year Bonds	31.02	34.29	47.71	34.8
5-10 Year Bonds	67.35	60.0	-----	-----
10+ Year Bonds	75.86	53.89	57.20	58.22
Moody's	-----	-----	36.91	42.18
McLeod, Young, Weir	39.86	30.22	-----	-----
Corporate Bonds	-----	-----	299.7	275.6

TABLE A.2, (continued)

	CANADIAN		U.S.	
Level of Test (Type I Error)	0.1	0.01	0.1	0.01
M CPI	25.78	21.56	23.82	17.75
Q CPI	45.42	32.51	40.18	15.96
Y CPI	43.13	37.6	34.95	26.4
Level CPI	32.00	31.75	32.00	24.11
RPCG	59.49	51.09	65.71	52.11
RPCM	27.09	16.98	36.91	24.11
ΔRPCG	130.19	125.67	52.62	47.27
ΔRPCM	42.47	28.436	42.80	27.42

Rejection regions for the test statistic are $S>44.32$ at the 10 per cent level, and $S>48.3$ at the one per cent level.

TABLE A.3

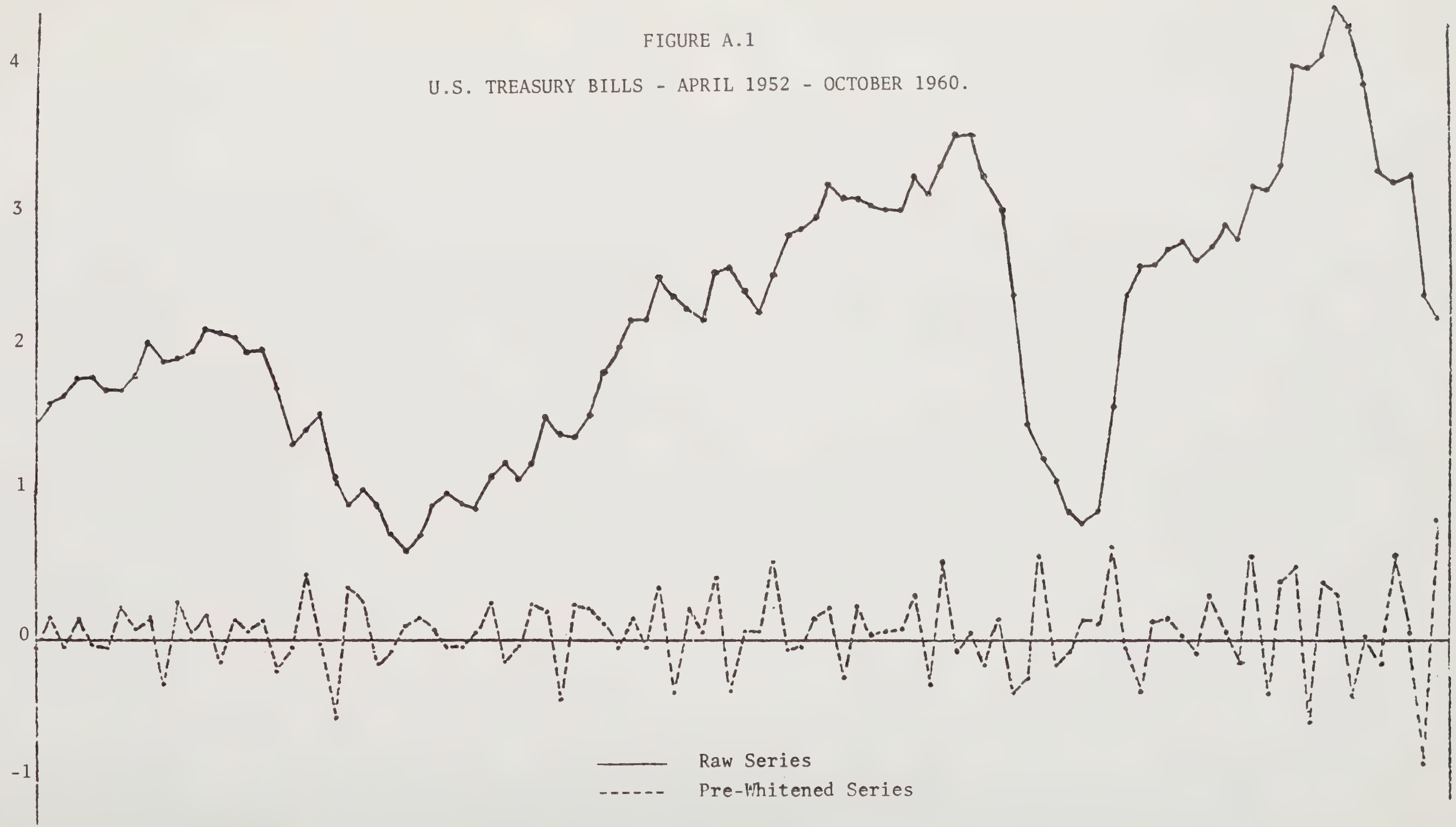
TEST OF STATIONARITY ASSUMPTIONS

TYPE OF TIME SERIES	CANADA	U. S.
Treasury Bills	-0.525	-0.478
9-12 Month Bonds	------	-0.456
1-3 Year Bonds	-0.295	------
3-5 Year Bonds	-0.181	-0.157
5-10 Year Bonds	-0.775	------
10+ Year Bonds	0.303	-0.166
McLeod, Young, Weir	-0.393	------
Corporate Bonds	------	1.207
Moody's	------	-0.217
RPCG	-0.318	-0.170
RPCM	-0.758	-0.163
MCPI	-0.016	-0.616
QCPI	0.200	-0.173
YCPI	-0.214	-0.569
Level CPI	-0.075	0.142

Critical region for rejection of stationarity is test statistic > 1.645 or < -1.645. All statistics are based on estimates using 110 observations and 25 Lags.

FIGURE A.1

U.S. TREASURY BILLS - APRIL 1952 - OCTOBER 1960.



APPENDIX B

CONCEPTS OF SPECTRUM ANALYSIS

This section of the Appendix contains definitions of the quantities central to multivariate spectrum estimation and a short discussion of the distribution of the various types of coherences obtained under the assumption that the original time series follow the multivariate normal distribution. We shall concentrate on sample estimates only, suppressing the usual distinction between sample and population values. We define the estimate of the cross spectrum as

$$g_{xy}(\lambda) = (2\pi)^{-1} \sum_{s=-m}^{m} K_{m,s} C_{xy,s} e^{-i\lambda s}$$

where

$$C_{xy,s} = T^{-1} \sum_{t=0}^{T-|s|} (x_t - \bar{x})(y_{t-s} - \bar{y})$$

is the maximum likelihood estimate of the lagged cross covariances,

λ indicates the frequency at which the cross spectrum is being estimated,

T is the total number of observations involved,

m the number of lags employed, is chosen by the analyst, and

$K_{m,s}$ is the lag window.

The Parzen lag window is used throughout this analysis because it insures positive spectrum estimates when used in combination with the maximum likelihood estimates of the lagged cross-covariances. The bivariate coherence function is defined as

$$R^2_{xy}(\lambda) = g_{xy}(\lambda)g_{xy}^*(\lambda)/[g_{xx}(\lambda)\ g_{yy}(\lambda)]\ \lambda\varepsilon\ [0,\pi]$$

where the asterisk indicates the conjugate of the complex valued cross-spectrum, and $g_{xx}(\lambda)$ is the spectrum of time series X, defined analogously to $g_{xy}(\lambda)$. The spectrum estimates are derived from the sample second movements of the time series, and associations with more familiar regression results are valid and enlightening.

Multivariate spectrum analysis also rests on estimates using sample second moments. It is now useful to treat Y as the endogenous variable and $\underset{\sim}{X}$ as a 1xp matrix of variables. Now, $\underset{\sim}{g}_{xx}(\lambda)$ is a pXp Hermitian matrix - that is, $\underset{\sim}{A}$ is Hermitian when $\underset{\sim}{A}^* = \underset{\sim}{A}$, the asterisks standing for the operation of conjugation of each element of $\underset{\sim}{A}$ followed by transposition of those elements. Also $\underset{\sim}{g}_{xy}(\lambda)$ is a pX1 matrix of complex elements. Of course, $\underset{\sim}{g}_{yy}(\lambda)$ remains a real 1X1 matrix.

The real valued multiple coherence function between Y and the p variables of $\underset{\sim}{X}$ is defined as:

$$R^2_{y.1,\ldots,p}(\lambda) = \underset{\sim}{g}_{xy}^*(\lambda)\ \underset{\sim}{g}_{xx}^{-1}(\lambda)\ \ \underset{\sim}{g}_{xy}(\lambda)\ /g_{yy}(\lambda),\ \lambda\varepsilon[0,\pi].$$

We will require two additional terms, the marginal and partial coherence functions. The partial coherence function is the coherence between Y and a subset of X, say the first q elements for definiteness, conditionalized on the last p-q elements of $\underset{\sim}{X}$. Partitioning $\underset{\sim}{g}_{xx}$ into four appropriate sub-

matrices, we have the representation:

$$g_{xx}(\lambda) = \begin{bmatrix} A_{11}(\lambda) & A_{12}(\lambda) \\ A_{21}(\lambda) & A_{22}(\lambda) \end{bmatrix} .$$

Partitioning $g_{xy}(\lambda)$ similarly we have

$$g_{xy}(\lambda) = \begin{bmatrix} B_1(\lambda) \\ B_2(\lambda) \end{bmatrix}$$

where $A_{11}(\lambda)$ is the qxp cross-spectral matrix for the first q elements of X and $B_1(\lambda)$ is the qx1 cross-spectral matrix of y and the first q element of X. We shall define the partial coherence function as:

$$R^2_{y.1,\ldots,q|q+1,\ldots,p}(\lambda) = g_{zy}(\lambda)\ g_{zz}^{-1}(\lambda)g^*_{zy}(\lambda)/g_{yy}(\lambda)$$

where $$g_{zz}(\lambda) = A_{11}(\lambda) - A_{12}(\lambda)A_{22}^{-1}(\lambda)A_{21}(\lambda),$$

and $$g_{zy}(\lambda) = B_1^*(\lambda) - A_{12}(\lambda)A_{22}^{-1}(\lambda)B_2(\lambda).$$

The marginal coherence function is defined as:

$$R^2_{y.1,\ldots,q}(\lambda) = B_1^*(\lambda)\ A_{11}^{-1}(\lambda)\ B_1(\lambda)/g_{yy}(\lambda).$$

The bivariate coherence defined earlier in this section and used to analyze the naive Fisherian hypothesis is the simplest type of marginal coherence function.

Under the assumption that Y and X follow the multivariate normal distribution, the density of each point on the coherence function is:

$$C(R|n,P,\gamma^2) = \frac{\Gamma(n)}{\Gamma(P-1)\Gamma(n-P+1)} (1-\gamma^2)^n R^{P-2} (1-R)^{n-p} F(n,n,P-1;\gamma^2;R),$$

where n is the effective degress of freedom, P is the effective number of records, γ^2 is the true coherence, R is the sample estimate of γ, and F(.,.;.;.) denotes the hypergeometric function. As stated earlier, n depends only on the value of M, the frequency involved, and the number of terms upon which the coherence has been conditionalized, if any, once the sample size and the lag window used in estimation are specified. The value of P is the number of independent variables directly considered plus one - that is, not including those variables upon which the particular coherence is conditionalized.

The density is quite simple when γ^2 is zero. Fortunately, this corresponds to the null hypothesis of no relationship between Y and X (or Y and some subset of $\tilde{X}$). Under the null hypothesis, the density collapses to

$$C(R|n,P,0) = \frac{\Gamma(n)}{\Gamma(P-1)\ \Gamma(n-P-1)} R^{P-2} (1-R)^{n-p}$$

Our summary test statistic is based on finding the mean, μ, and standard deviation, σ, of R given $\gamma^2 = 0$. Choosing L roughly independent estimates of γ^2 on the interval $[0,\pi]$, we form a weighted sum of the $R(\lambda_i)$. It follows from a central limit theorem that the test statistic,

$$L^{-\frac{1}{2}} \sum_{i=1}^{L} [R(\lambda_i) - \mu_i]/\sigma_i ,$$

is asymtotically distributed N(0,1). Both the mean and the standard deviation of $R(\lambda i)$ have very simple forms:

$$\mu_1^i = \mu^i = \frac{\Gamma(\delta_i n)\Gamma(P-1/2)}{\Gamma(P-1)\Gamma(\delta_i n+1/2)}$$

and

$$\sigma_i = \sqrt{\frac{\Gamma(P)}{\delta_i n\Gamma(P-1)} - (\mu^i)^2}$$

where $$\delta_i = \begin{cases} \frac{1}{2}, \text{ if } \lambda_i = 0,\pi \\ 1, \text{ otherwise} \end{cases}$$

The density and cumulative distribution function for small n and P=2 has been tabled in (2), and the cumulative distribution for small n and P is tabled in (1).

The non-parametric test of the null hypothesis depends only on the first moment, μ^1, derived above. We take every other estimated coherence and treat it as if it were an independent trial from the Binomial distribution $B(\frac{1}{2},L)$. By scoring a one for each estimated coherence which is greater than μ^1 and a 0 for each that is less than μ^1, the sum of the scores can be approximated by the normal distribution $N(L/2,L/4)$ under the null hypothesis. The value recorded in Table B.1 are standardized test statistics and should be compared to the standard normal distribution under the null hypothesis.

The invariance of the coherence function with respect to linear transformations of the individual underlying time series is easily established for an infinite sample. Consider the bivariate case

$$Z_t = \sum_{u=-r}^{s} b_u x_{t-u},$$

where b's are finite real constants, s and r are finite integers, and x_{t-u} is a time series from a covariance stationary ergodic process. The covariance function of the process Z_t is

$$C_{ZZ,\tau} = E(Z_t Z_{t+\tau}) = \sum_{u,v=-r}^{s} b_u b_v C_{xx,\tau+u-v}$$

The spectrum of this process is

$$g_{ZZ}(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} \sum_{u,v=-r}^{s} b_u b_v C_{xx,\tau-u+v} e^{-i\lambda\tau}$$

$$= (2\pi)^{-1} \sum_{u=-r}^{s} b_u e^{-i\lambda u} \sum_{v=-r}^{s} b_v e^{i\lambda v} \sum_{\tau=-\infty}^{\infty} C_{xx,\tau-u+v} e^{-i\lambda(\tau-u+v)}$$

$$= |B(\lambda)|^2 g_{xx}(\lambda).$$

The term B( ) is called the frequency response function, or transfer function associated with the linear transformation defined by the b's. Note that linear transformations in the time domain amount to multiplicative transformations in the frequency domain. The interchanging and separations of the summations above are valid as long as the individual series are absolutely convergent. This is insured when the b's and s and r are finite and the X process is ergodic.

Without loss of generality, we shall consider the bivariate coherence between X and Z:

$$R^2_{zx}(\lambda) = |g_{xz}(\lambda)|^2 / [g_{xx}(\lambda)\, g_{zz}(\lambda)].$$

But the cross-spectrum between X and Z is

$$g_{xz}(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} C_{xz,\tau}\, e^{-i\lambda\tau}$$

$$= (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} \sum_{u=-r}^{s} b_u C_{xx,\tau+u}\, e^{-i\lambda\tau}$$

$$= (2\pi)^{-1} \sum_{u=-r}^{s} b_u e^{i\lambda u} \sum_{\tau=-\infty}^{\infty} C_{xx,\tau+u}\, e^{-i\lambda(\tau+u)}$$

$$= B(\lambda)\, g_{xx}(\lambda) .$$

Thus, $R^2_{xz}(\lambda) = B(\lambda)\, B^*(\lambda)\, g^2_{xx}(\lambda)/|B(\lambda)|^2\, g_{xx}(\lambda) = 1$ for all λ. Of course, the coherence between a series and a linear transformation of itself is one. The generalization of other functions follows immediately because we have the condition that the transfer function of filters, which are sensible from an economic point of view - ones which do not mix the time series together - is a diagonal matrix. Finally, it must be mentioned that the invariance property of the coherence functions holds only approximately for finite samples.

We have made repeated use of the approximate invariance of the coherence functions with respect to linear transformations of the data which do not mix the time series together. This property plays an important part in the Hannan esti-

mation routine because it is used to provide an estimate of the residual spectrum (See Appendix C). We have also used it directly in the preceding analysis to appraise the validity of whole classes of models - all variants of a model with a given endogenous variable and a specified set of exogenous variables. For a more detailed discussion see Fishman [9].

TABLE B.1: SELECTED NON-PARAMETRIC TEST RESULTS

	MCPI	QCPI	YCPI	Level CPI	Partial Coherence with MCPI Conditionalized on RPCG and RPCM.	Multiple Coherence with MCPI, RPCG, and RPCM.
U.S. Treasury Bills	-0.23	-0.23	-2.06	+0.23	-2.51	+1.15
U.S. 9-12 mo. Bonds	-0.23	-1.61	-2.06	+1.15	-2.51	+0.23
U.S. 3-5 yr. Bonds	+0.69	-0.69	-1.15	+1.61	-2.06	+0.69
U.S. 10+ yr. Bonds	-0.23	-0.23	-0.69	+2.06	-1.61	+0.23
Corp. Bonds	-1.15	-2.06	-2.06	+0.23	-3.44	-0.69
Moody's	+0.69	-0.69	-0.23	+1.61	-1.15	+2.98
Can. Treasury Bills	-0.69	-0.69	-1.61	-0.69	-2.98	-0.69
Can. 1-3 yr. Bonds	-0.23	-1.15	-0.69	-0.23	-2.51	-2.51
Can. 3-5 yr. Bonds	-0.69	-1.61	-1.61	-0.23	-2.98	-2.98
Can. 5-10 yr. Bonds	+0.23	-1.15	+0.23	+0.69	-2.98	-1.15
Can. 10+ yr. Bonds	-1.15	-0.69	+0.23	+1.15	-1.61	-1.61
McLeod, Young, Weir	-0.23	-0.23	-0.23	+0.69	-1.15	-2.51

Critical region for test of no relationship is test statistic $\ngtr$ 1.645.

APPENDIX C

THE HANNAN ESTIMATION ROUTINE

We present a brief introduction to the Hannan estimation routine for two time series in this section. While the routine can be extended to any number of time series, it becomes numerically complicated and difficult to apply without prior specification of the lag structure.

Let us discuss the simplest model first, following the notation and arguments from Hannan's seminal article [18]. Suppose that Y and X are two covariance stationary, ergodic processes which we observe at discrete, equally-spaced points in time. Suppose further that Y is a linear function of the contemporaneous value of X plus an error term. Then

$$y_t = Bx_t - e_t \qquad \text{(C-1)} .$$

The ordinary least square (O.L.S.) estimates of B is

$$\left(\sum_{t=1}^{T} x_t x_t\right)^{-1} \sum_{t=1}^{T} x_t y_t .$$

We know that this estimator, under the assumption that the e_t's are i.i.d., possesses many desirable properties. How-

ever, it is often the case in empirical work that the e's are not independent. If the spectrum of the residual term exists and is not constant, estimation with O.L.S. will not be efficient. Multiplying by x_t and applying the Fourier transform to both sides of equation (C-1), we obtain

$$g_{xy}(\lambda) = Bg_{xx}(\lambda) + g_{xe}(\lambda), \quad \lambda\varepsilon[0,\pi] \quad .$$

We require the residual process to be independent of the exogenous variable by adding the condition that $g_{xe}(\lambda) = 0$ for all λ. This is analogous to requiring that the error be orthogonal to the exogenous variables in O.L.S. estimation. Testing this assumption with the summary test statistic for the coherence function outlined in the preceding appendix provides a criteria for deciding when the model has been specified correctly.

The time domain representation of the equation indicates that B is a constant. However, we estimate the spectrum and cross-spectrum at M + 1 discrete points and thus, have M + 1 estimates of B, b(K), where

$$b(K) = \hat{g}_{xx}^{-1}(\pi K/M)\ \hat{g}_{xy}(\pi K/M), \ K = 0,\ldots,M.$$

We are faced with the question of how to obtain a single estimate of B. If we assume the spectrum estimates are roughly independent, the approximate optimal weighting pattern is

$$b = (2M)^{-1} \sum_{K=-M+1}^{M} \hat{g}_{ee}^{-1}(\pi K/M)\ \hat{g}_{xy}(\pi K/M)\ x$$

$$[(2M)^{-1} \sum_{K=-M+1}^{M} \hat{g}_{ee}^{-1}(\pi K/M)\ \hat{g}_{xx}(\pi K/M)]^{-1}$$

The asymptotic variance of this estimator is the variance of the B.L.U. estimator,

$$[T(2\pi)^{-1} \sum_{\pi}^{\pi} g_{xx}(\lambda)\ g_{ee}^{-1}(\lambda)\ d\lambda]^{-1} \quad .$$

The central problem encountered in handling residuals which do not conform to the usual assumptions is determining the salient features of the divergent behavior. It is often

practical to assume the residuals follow a simple pattern such as first order Markov. This procedure is not adequate when the data and residuals have seasonal or other cyclical components of importance. There are several procedures for obtaining estimates of $g_{ee}(\lambda)$, but we chose to adopt a method suggested by Hannan [18] and Fishman [9]. These authors note that a consistent estimate of $g_{ee}(\lambda)$ can be obtained directly from the estimates of the bivariate coherence function $\hat{R}_{xy}$, and the spectrum of the dependent variable, $\hat{g}_{yy}(\lambda)$. We have adopted this approach, estimating the residual spectrum by $\{(1-\hat{R}^2_{yx}(\lambda)\}\hat{g}_{yy}(\lambda)\}$.

Now consider the full distributed lead-lag model,

$$Y_t = \sum_{i=-r}^{s} B_i X_{t-i} + e_t.$$

If we knew s and r, theoretically we could get an estimate of the variance covariance matrix of residuals by using the O.L.S. estimators of e_t as a first step, and deriving an estimate of the variance-covariance matrix of residuals from them. However, we face the problem that the corner elements of that estimate rely on only one product, $\hat{e}_1\hat{e}_T$. Indeed, adding more observations does not rectify this problem, as the size of the variance-covariance matrix increases with the number of observations. Of course, the fundamental problem is that we do not know r and s. Hence, in addition to having trouble with the extreme elements of Σ, we cannot obtain consistent estimates of its elements. However, we have the relationship:

$$\int_{\tau=-\infty}^{\infty} C_{ee,\tau} e^{-i\lambda\tau} d\tau = g_{ee}(\lambda)$$

and the consistent estimate of $g_{ee}(\lambda)$. Using the inverse Fourier transformation we can obtain

$$\int_{\lambda=-\pi}^{\pi} g_{ee}(\lambda)\, e^{i\lambda\tau}\, d\lambda = C_{ee,\tau}.$$

Translating this into our finite sample context, we can estimate the elements of the variance-covariance matrix by

$$\hat{C}_{ee,\tau} = (2M)^{-1} \sum_{K=-M+1}^{M} \hat{g}_{ee}(K\pi/M) e^{i(K\pi/M)\tau} .$$

These elements could be put into a TXT matrix and we could perform a type of generalized least squares. However, the variance-covariance matrix in time series analyses has a very special character when covariance stationarity is assumed. The matrix is called a Toeplitz form and is symmetric with all of the elements on any diagonal identical. That is,

$$\Sigma = \begin{bmatrix} C_{ee,0} & C_{ee,1} & \cdot & \cdot & \cdot & \cdot & \cdot & C_{ee,T-1} \\ C_{ee,1} & C_{ee,0} & & & & & & \cdot \\ \cdot & \cdot & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & \cdot & C_{ee,1} \\ C_{ee,T-1} & \cdot & \cdot & \cdot & \cdot & & C_{ee,1} & C_{ee,0} \end{bmatrix}$$

By using several limited arguments, maximum likelihood estimates of the b's may be obtained by solving s-r-1 linear equations in s-r-1 unknown. The form of the equation is $\hat{\beta} = (\hat{B}_s \ldots \hat{B}_0 \ldots \hat{B}_r)'$ where

$$\hat{\underset{\sim}{\beta}} = \hat{\underset{\sim}{H}}^{-1}\ \hat{\underset{\sim}{D}}\ ,$$

$$\hat{h}_{\ell s} = (2M)^{-1} \sum_{j=M+1}^{M} \hat{g}_{ee}^{\ -1}(\lambda_j)\hat{g}_{xx}(\lambda_j)e^{i\lambda_j(\ell-\nu)}, \ell,\nu = 1\ldots,s+r+1$$

is the entry in the row ℓ and columns s of $\hat{H}$, and

$$\hat{d}_\ell = (2M)^{-1} \sum_{j=M+1}^{M} \hat{g}_{ee}^{\ -1}(\lambda_j)\hat{g}_{xy}(\lambda_j)e^{i\lambda_j\ell}\ , 1 = 1,\ldots,s+r+1$$

is the entry row ℓ of $\hat{D}$.

The asymptotic variance-covariance matrix of $\hat{B}$ is $T^{-1}\hat{H}^{-1}$.

A more detailed discussion of the procedure can be found in Fishman [9].

The signal-to-noise ratio is defined as

$$\hat{g}_{xx}(\lambda_j)/\hat{g}_{ee}(\lambda_j).$$

The denominator is called the noise term and is the value of the residual spectrum at frequency λ_j. The numerator is called the signal and is the value of the spectrum of the independent variable at the same frequency. The inverse of this ratio is proportioned to the variance of the estimate b(K) in model (1). We can obtain a minimum variance estimate of B in model (1) by combining roughly independent estimates using weights proportioned to the inverse of the individual variances. We obtain,

$$b = \sum_{K=-M+1}^{M} \frac{b(K) \quad \hat{g}_{xx}(K\pi/M)/\hat{g}_{ee}(K\pi/M)}{\sum_{K=-M+1}^{M} \hat{g}_{xx}(K\pi/M)/\hat{g}_{ee}(K\pi/M)} .$$

This is, of course, equivalent to the weighting pattern presented above. Note that the signal-to-noise ratio at each frequency plays a central role in this estimation routine because its inverse is proportional to the variance of the estimate of the regression coefficient at the same frequency.

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THE MONEY SUPPLY AND THE RATE OF INFLATION

by

J.L. Carr

INTRODUCTION

Before 1935 there was considerable agreement among economists as to the principal cause of inflation. Most economists adhered to the quantity theory. They believed an increase in the money supply would be followed (not necessarily instantaneously) by an increase in the price level. With the publication of Keynes' "General Theory" the quantity theory fell into disrepute and was rejected by an overwhelming majority of economists. During the 1930s the primary economic problem was not one of inflation but one of unemployment. As a consequence when the money supply was rejected as a prime determinant of the absolute price level, there seemed to be no great need to find an alternative theory of inflation. In fact, no alternative was found. In the Keynesian model all variables were expressed in wage-units and hence the model explained real variables. The absolute price level was treated as an exogenous variable. In the short run, when aggregate demand increased, the Keynesian model had quantity adjustments and no price adjustments. In the short run the price level could be treated as fixed in the Keynesian model.[1] One of the problems with the Keynesian model is its failure to analyze inflation. One possible way out of this dilemma is to assume that

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[1] For a discussion of the rigid absolute price level in the Keynesian model see M. Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, March 1970.

up to full employment, all changes in aggregate demand will be reflected in real output and beyond that point all changes in aggregate demand will be reflected in prices. In fact this is what Keynes argued in one section of the General Theory, that under certain assumptions ".....an increase in the quantity of money will have no effect whatever on prices, so long as there is any unemployment, and that employment will increase in exact proportion to any increase in effective demand brought about by the increase in the quantity of money; whilst as soon as full employment is reached, it will thence-forward be the wage-unit and prices which will increase in exact proportion to the increase in effective demand."[2] Thus it can be argued that ultimately Keynes uses the quantity theory as his explanation of inflation.[3] But the above theory leaves unexplained how inflation and unemployment can co-exist. As inflation became more of a problem during the 1950s and as it was observed that inflation and unemployment did sometimes occur together, economists developed theories of inflation capable of explaining the co-existence of inflation and unemployment. These theories tended to explain inflation as a "cost" or "price" push.[4] These theories tended to attribute each bout of inflation to something specific about that inflation, and as such they are not really theories at all, since they cannot be applied in general or cannot be used to predict when the next bout of inflation will occur (because they cannot predict when the next "price" or "cost" push will occur).

The primary purpose of this paper will be to investigate the influences of money on the absolute price level. The second section will present a theoretical model which outlines the influence of money of the price level. The model outlined in section two is essentially a dynamic version of

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[2] John Maynard Keynes, The General Theory of Employment, Interests and Money, London, Macmillan 1964, p. 295.

[3] This should not be too surprising since Keynes prior to the general theory adhered to the quantity theory. See J.M. Keynes, Tract on Monetary Reform, London, 1924.

[4] For example, see J.K. Galbraith, "Administered Prices and Monetary-Fiscal Policy", reprinted in Money and Economic Activity, edited by L.S. Ritter, Boston, Houghton Mifflin Co., 1961.

the quantity theory. Section three will present the empirical results of the estimation of the reduced-form equation explaining the rate of inflation. Section four will look at the question of the appropriate definition of the money supply and empirical results using different definitions of money will be presented. Section Five will present the conclusions obtained from the empirical results.

THE MODEL[5]

Definition of Symbols

M_t : money supply at time t

y_t : real income at time t

V_t^d : desired velocity at time t

V_t : actual velocity at time t

i_t : nominal interest rate at time t

The model will be estimated in logarithmic form and therefore the first difference in the logarithm of a variable will be used to approximate the percentage change in that variable.

$\Delta\log M_t$: percentage change in the money supply from t-1 to t

$\Delta\log y_t$: percentage change in real income from t-1 to t

$\Delta\log P_t$: percentage change in the absolute price level from t-1 to t

$\Delta\log M_t^e$: expected percentage change in the money supply where expectations are formed at time t

$\Delta\log y_t^e$: expected percentage change in real income where expectations are formed at time t

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[5] The model employed here is an adaptation of part of the model in J.L. Carr, "A Dynamic Monetary Model of Business Fluctuations". Ph.D. dissertation, University of Chicago, 1971.

$\Delta \log P^e_t$: expected percentage change in prices where expectations are formed at time t

Structural Equations of the Model

$$\log V^d_t = a_0 + a_1 \log i_t + a_2 \log y_t + a_3 [\Delta \log P_t - \Delta \log P^e_{t-1}] \qquad (1).[6]$$

This equation has desired velocity as a function of three variables, the nominal interest rate, real income and the difference between the actual and expected rate of inflation. The higher the nominal interest rate, the greater is the opportunity cost of holding cash balances, and as a consequence the less will be desired real cash balances and the higher will be desired velocity. Therefore, *a priori*, one would expect a_1 to be a positive.

The real income variable represents an income effect. The sign of a_2 can not be determined *a priori*. For, as an economy grows through its early stages of development, the demand for money is increased from two possible sources. As income increases, more money is demanded to handle the increase in transactions. Also as income increases money replaces barter for a larger and larger section of the economy. In rural areas barter is the main method of local exchange even when money is used extensively in the urban areas. With growth, industrialization and urbanization, increases in income cause more money to be demanded for normal transaction pur-

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[6]Instead of using the concept of desired velocity one could equally well have used the concept of the desired number of weeks income the community wishes to hold in real cash balances, k^d, where $k^d \equiv 1/V^d$. For certain purposes of exposition, it is perhaps more convenient to think in terms of k^d rather than V^d. Similarly one could transform (1) into a demand for real cash balances equation. In this case (1) becomes:

$$\log (M/P)^d_t = -a_0 - a_1 \log i_t + (1 - a_2) \log Y_t - a_3 [\Delta \log P_t - \Delta \log P^e_{t-1}].$$

poses and also for transactions which were previously being conducted by barter. In the early stages of growth an increase in income probably implies higher real cash balances and lower velocities. Eventually the process of urbanization slows down and the whole economy is in fact on a money exchange basis. Past this point, increases in income result in increased transaction demand but the percentage increase in transaction demand for money may not be as great as the percentage increase in income and so velocity may start to rise. Therefore the exact effect of income on velocity cannot be determined a priori.

It should be noted that desired velocity is an aggregate concept. It is a weighted average of individual velocities. If there is income redistribution, aggregate desired velocity can change while each individual's desired velocity remains the same.[7] Now if income is redistributed from people with high real cash balances (i.e. high k) to people with low real cash balances, aggregate desired velocity will increase. When workers negotiate contracts with their employers they take into consideration the expected price change. Similarly when lenders lend they too take into account an expected price change. When the actual price change is different from the expected some parties will gain while others will lose. Unexpected inflation redistributes income; with employers gaining at the expense of employees and debtors at the expense of creditors. This redistribution of income is only temporary, since eventually employees will revise their expectations about price change and include these new expectations in their contract negotiations and lenders also will revise their expectations about price change and will demand a higher nominal rate of interest. If a part of the transitory income gained or lost during the temporary redistribution of income, produced by unexpected price changes, goes

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[7]This is easily seen using k the inverse of velocity. If there are n individuals who hold cash balances then the total desired nominal quantity of money is

$$M^d = k_1y_1 + k_2y_2 + \ . \ . \ . + k_ny_n \text{ where } k_i = \frac{1}{V_i}$$

$$\text{thus } K = \frac{M^d}{Y} = k_1\left[\frac{Y_1}{Y}\right] + k_2\left[\frac{Y_2}{Y}\right] + \ . \ . \ . + k_n\left[\frac{Y_n}{Y}\right] \text{where } K = \frac{1}{V^D} .$$

into or comes out of cash balances and, in addition the people who benefit have, on average, a different desired velocity than those who lose, then aggregate desired velocity will change as a result of an unexpected price change. The greater the unexpected price change, the greater is the redistribution of income and hence the greater is the effect on desired velocity. There is perhaps a presumption in the literature as to the direction of change in desired velocity when unexpected inflation occurs. As stated earlier, unexpected inflation redistributes income from employee to employer and also from creditor to debtor. To the extent that business firms are net debtors, unexpected inflation redistributes income in favor of firms, and if firms hold lower real cash balances then households (i.e. if households owing firms maintain on average, lower cash balances in relation to their income than households not owing firms) then unexpected inflation increases desired velocity. There is some presumption that a_3 is positive. How important this redistribution term is, is not known a priori. Since this term is generally not included in most demand for money functions, the model will be estimated both including and excluding this redistribution term.

$$\log V_t \equiv \log P_t + \log y_t - \log M_t \qquad (2).$$

This is a definition of actual velocity.[8] Being a definition this identity holds at all times and for all values of M,P, and y.

$$\Delta\log P_t = \alpha[\log V^d_{t-1} - \log V_{t-1}]$$
$$+ [\Delta\log M^e_{t-1} + \Delta\log V^e_{t-1} - \Delta\log y^e_{t-1}] \text{ where } 0 \leq \alpha \leq 1 \qquad (3).$$

This is the price change equation. It says whenever actual velocity differs from desired velocity, the rate of increase in prices change, and the percentage change in price is a function of the difference between desired and actual velocity. That is, it is assumed that prices do not adjust instantaneously, but take time to adjust.[9] The speed of adjustment is measured by α. The greater is α, the greater is the speed of adjustment of prices to changes in

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[8]Re-written in another form this is what is known as the quantity identity.

the money supply. If $\alpha=0$, prices would never adjust to changes in the money supply and if $\alpha=1$ prices would instantaneously adjust (i.e. within one time period) to changes in the money supply. It would seem reasonable to expect that α would be a function of the variability in the rate of inflation. For with a stable rate of change of prices more people will be willing to make contracts in nominal terms for a considerable period of time. For example, if all workers knew that absolute prices would increase at the rate of three percent a year indefinitely they would not hesitate to sign wage contracts for one, two or three years.[10] The more of these contracts that exist, the stickier will be the price level (i.e. the stickier will be the rate of inflation) and hence the smaller will be α. On the contrary, if the rate of change of prices is quite variable, people will hesitate to make such contracts because of the risk of capital loss and prices will adjust fairly readily to unexpected changes in the money supply. If we compare the Canadian economy to an economy in which the rate of change of prices has been more variable (what matters here is not whether the rate of inflation is high or low but how variable it is) we would expect to find a lower α for the Canadian economy. Also if over time, in the Canadian economy, the variability of the rate of inflation increases one would also expect α to increase.[11] For the purposes of this model, we assume α is a constant.

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[9] The reasons prices do not adjust instantaneously to changes in demand conditions is a subject which has currently received a fair amount of attention in the literature concerning the microeconomics of inflation. In this literature the existence of search, information and transaction costs (of distinguishing between changes in real and changes in nominal demand) result in prices which do not adjust instantaneously. For a summary of this literature see, E. Phelps, et. al., Microeconomic Foundations of Employment and Inflation Theory New York, W.W. Norton and Co., 1970, pp. 1-27.

[10] It should be noted that this analysis ignores the risk associated with changes in relative wages. The risk of variability of absolute price is an additional risk in making contracts in nominal terms.

[11] This means the greater the variability of the rate of inflation, the more flexible are prices and the greater is α and the greater will be the proportion of any change in the rate of growth of the money supply that will go into prices (in the short run).

Equation (3) says the greater is the difference between actual and desired velocity, the more will prices have to adjust. When desired velocity is greater than actual, people's actual real cash balances will be greater than they desire. They will try to get rid of this excess cash and in so doing will drive prices up.[12] Now when actual velocity is equal to desired velocity and the model is in equilibrium with expected quantities equal to their actual magnitudes then (3) says the rate of price change will be equal to the rate of change of money plus the rate of change of velocity minus the rate of change of real income. In equilibrium we have the simple quantity theory result.

Ignoring feedback effects, (3) says that there are two effects on the rate of change of prices when the rate of change of money increases. One will be that desired velocity will be different from actual and this causes the rate of change of prices to rise. This effect operates at once. The other effect is our second term,

$$[\Delta \log M^e_{t-1} + \Delta \log V^e_{t-1} - \Delta \log y^e_{t-1}]$$

which <u>initially</u> has no affect on the movement of the rate of inflation to its new equilibrium level but which will gradually rise and cause the rate of change of prices to move gradually from its old equilibrium level to its new equilibrium level.

Equation (3) says that prices adjust instantaneously to perfectly expected changes in money (or income or velocity for that matter) but that they adjust with a lag to unexpected (or imperfectly expected) changes in money. Since

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12 The above analysis says nothing about the transmission mechanism whereby changes in money bring about changes in the price level. In the current state of economic knowledge, we know very little about the actual transmission mechanism. In M. Friedman and A. Schwartz, "Money and Business Cycles", <u>Supplement to Review of Economics and Statistics</u>, (February 1963), there is a very sketchy outline of the transmission mechanism whereby people trying to get rid of excess cash balances first cause financial asset prices to rise and then the prices of real assets and finally the prices of service flows.

we will be defining expectations so expected variables conform to the simple quantity identity of (2), equation (3) can be re-written as

$$\Delta \log P_t = \alpha[\log V^d_{t-1} - \log V_{t-1}] + \Delta \log P^e_{t-1} \quad (3).$$

It will be noted that unemployment (or excess capacity) and foreign prices (in particular U.S. prices) do not appear as independent variables in equation (3). These two variables are often considered as the main causes of inflation, yet they are omitted in this discussion of inflation. Why? Let us first look at the unemployment effect. It will be argued here that inflation and unemployment are often influenced by a common stimulus and that this is the reason why an empirical association between these variables is observed. It will be argued that unemployment does not exert any <u>direct</u> influence on the rate of inflation. In our model if unemployment were to be made an endogenous variable it would be handled as such

$$\Delta \log y_t - \Delta \log y^e_{t-1} = f(\Delta \log M_t - \Delta \log M^e_{t-1}, \text{ real factors}^{13} \quad (12)$$

$$\text{unemployment rate} - f(y - y^e) \quad (13).$$

In such a model unemployment would be affected by both real and monetary magnitudes and in equilibrium unemployment would be at its natural level. If the monetary authorities were to increase the rate of growth of the money supply, the rate of growth of real income would increase, real income would grow and the unemployment rate would fall. But these effects would be temporary.[14]

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[13] For a discussion of this equation, see J.L. Carr, <u>op</u>. <u>cit</u>. pp. 17-21. The rationale for unexpected monetary changes influencing the rate of growth of output is as follows. An unexpected increase in monetary demand will cause firms to be faced with increased demands but firms will not know if these are increases in nominal demands or increases in real demands. Faced with this uncertainty, firms will to some extent expand output. With expanded output, unemployment will fall. For a summary of a number of variants of this argument see E. Phelps, et. al., <u>op</u>. <u>cit</u>.

Unemployment in the model is a "disequilibrium" phenomenon. For example, suppose we are initially in equilibrium with all actual magnitudes equal to their expected values. Now decrease the rate of growth of money so that $\Delta \log M < \Delta \log M^e$ and therefore $V > V^d$. From equation (12) the unexpected decrease in $\Delta \log M$ will have a depressing effect on $\Delta \log y$, cause y to be less than y^e, and unemployment results. Since $V > V^d$, from equation (3) of the model we see that the rate of inflation will fall. Therefore unexpected monetary decreases cause both unemployment <u>and</u> a lowering of the rate of inflation. It is <u>not</u> the unemployment which is causing a reduction in the rate of inflation. If unemployment in the real world was mainly due to monetary shocks then one would expect a negative correlation between the rate of inflation and the rate of unemployment.[15] I believe that because empiricists have found this negative relationship between the rate of inflation and the rate of unemployment they have incorrectly concluded that the unemployment rate is a prime determinant of the rate of inflation and as a consequence have built models with unemployment a prime determinant of the rate of inflation to give this empirical relationship a theoretical foundation.

What should the effect of changes in the foreign price level[16] be on the domestic price level? To what extent can a country import inflation? The answer to these questions depends on the type of exchange rate system used. If the exchange rate is perfectly flexible, an increase in the foreign price level will be reflected in an offsetting change in the exchange rate and thus there will be no effect on the price of foreign goods denominated in Canadian dollars. The price level of Canadian goods denominated in foreign dollars

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[14] See M. Friedman, "The Role of Monetary Policy", <u>American Economic Review</u>, March 1968.

[15] If unemployment was mainly due to real shocks (e.g. bad harvests in a predominantly agricultural economy) then the above theory would predict a positive simple correlation coefficient between inflation and unemployment.

[16] Really the argument should be made in terms of the price level of tradeable foreign goods. For purposes of expositiion it will be assumed these two price levels move together.

will rise by as much as the rise in price of foreign goods (due to the fall in the exchange rate) and therefore there will be no increase in demand for Canadian goods and as a consequence no change in the price level of Canadian goods. Under a flexible exchange rate system there seems little reason to consider foreign inflation as a determinant of domestic inflation.

Suppose now, the fixed exchange rate system existed and the foreign price level increased. With increased foreign prices, Canadian goods will become more attractive to foreigners, thus increasing Canadian exports, and foreign goods will become less attractive to Canadians, thus decreasing Canadian imports. If we assume that the exchange rate was initially in equilibrium, then with the rise in the foreign price level, the fixed exchange rate will be out of equilibrium with the price of the Canadian dollar (in terms of foreign currency) below its equilibrium value. The fixed rate is maintained below its equilibrium value by the willingness of the Canadian government to hold foreign currency and exchange Canadian dollars for foreign currency (in order to satisfy the excess demand for Canadian dollars at the fixed exchange rate). The operations of the Canadian government in the foreign exchange market will increase the money supply.[17] The increase in the money supply will increase the domestic price level and restore external balance. Since the money supply is included in equation (3) there is no need to have the foreign price level in this equation. What the above analysis argues is that under a fixed exchange rate regime the foreign price level is one of the prime determinants of the money supply. Under fixed exchange rates, the primary influence of the foreign price level is not its direct effects on domestic prices but its effects (and constraints) it imposes on domestic monetary and fiscal policy.

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[17]It should be noted that the Bank of Canada can "sterilize" the effects of changes in foreign exchange reserves on the money supply. Such operations cannot continue for long periods of time (especially when the country concerned is running a balance of payments deficit). However, sterilization may result, in the domestic price level rising, in the short run, without any prior increase in the domestic money supply.

Solution of the Model

Substituting (1) into (3) we get

$$\Delta\log P_t - \Delta\log P^e_{t-1} + \alpha a_0 + \alpha a_1 \log i_{t-1} + \alpha a_2 \log y_{t-1}$$

$$- \alpha \log V_{t-1} + \alpha a_3 [\Delta\log P_{t-1} - \Delta\log P^e_{t-2}] \quad (5).$$

If the redistribution term is left out of (1) [i.e. $a_3 = 0$] then

$$\Delta\log P_t - \Delta\log P^e_{t-1} = \alpha a_0 + \alpha a_1 \log i_{t-1} + \alpha a_2 \log y_{t-1}$$

$$- \alpha \log V_{t-1} \quad (5A).$$

Both (5) and (5A) have $\Delta\log P_t$ as a function of predetermined variables only. Hence if one could observe an expected rate of price change variable both equations could be estimated directly using ordinary least-squares. However the expected rate of price change is not an unobservable variable. Therefore in order to estimate (5), a theory of price expectations must be added to the basic model. Three different solutions to this problem were posed.

(A) One solution is to use an expected rate of inflation series which has been generated elsewhere. One such series has been generated by myself and L.B. Smith.[18] In that paper the following model was run for Canadian data,

$$i_t = b_0 + b_1 [\Delta\log M_t - \Delta\log M^e_{t-1}] + b_2[\log P^e_t].$$

The $\Delta\log P^e_t$ variable represented the Fisherian effect which when added to the real interest rate gave the nominal interest rate. The $\Delta\log P^e_t$ variable was calculated by assuming that $\Delta\log P^e_t$ was some weighted moving average of past rates of inflation. The Almon[19] technique was used to calculate the weights. With the weights known, an expected rate of

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18 See J.L. Carr and L.B. Smith, "Money Supply, Interest Rates and the Yield Curve", Journal of Money, Credit and Banking, August 1972.

19 S. Almon, "The Distributed Lag Between Capital Appropriations and Expenditures", Econometrica. August 1965.

inflation series was generated. This series was used in the estimation of (5) and (5A).

(B) Another solution is to use the Almon technique directly on equations (5) and (5A).
Re-writing (5) and (5A), (5) is

$$\Delta \log P_t = \alpha a_0 + \alpha a_1 \log i_{t-1} + \alpha a_2 \log y_{t-1} + \alpha a_3 \Delta \log P_{t-1} - \alpha \log V_{t-1} + \Delta \log P^e_{t-1} - \alpha a_3 \Delta \log P^e_{t-2} \qquad (5)'.$$

(5A) is

$$\Delta \log P_t = \alpha a_0 + \alpha a_1 \log i_{t-1} + \alpha a_2 \log y_{t-1} - \alpha \log V_{t-1} + \Delta \log P^e_{t-1} \qquad (5A)'.$$

With the Almon technique it is assumed that

$$\Delta \log P^e_t = \sum_{i=0}^{n} w_i \Delta \log P_{t-i} \text{ and } w_i = a_0 + a_1 i + \ldots a_q i^q .$$

Since $\Delta \log P^e_t$ is a function of past values of $\Delta \log P$ it is very likely that $\Delta \log P^e_{t-1}$ and $\Delta \log P^e_{t-2}$ will be highly collinear. Therefore it is likely that problems of extreme multicollinearity will result if (5)′ is estimated using Almon variables. Instead (5)′ can be simplified if the $w_0 \Delta \log P_{t-1}$ term of $\Delta \log P^e_{t-1}$ is incorporated with $\alpha a_3 \Delta \log P_{t-1}$ and the rest of the terms of $\Delta \log P^e_{t-1}$ incorporated with $- \alpha a_3 \Delta \log P^e_{t-2}$.

With this simplification (5)′ becomes

$$\Delta \log P_t = \alpha a_0 + \alpha a_1 \log i_{t-1} + \alpha a_2 \log y_{t-1} + (\alpha a_3 + w_0) \Delta \log P_{t-1} - \alpha \log V_{t-1} - \alpha a_3 \sum_{i=0}^{n} w_i' \Delta \log P_{t-(i+2)} \qquad (5)''$$

Equations (5)′′ and (5A)′ can now be estimated using Almon variables.

(C) A third solution is to assume some adaptive mechanism in the formation of expectations. If the actual rate of inflation is larger than is expected, then the expected rate of inflation is revised upwards,

$$\Delta\log P^e_t - \Delta\log P^e_{t-1} = w[\Delta\log P_t - \Delta\log P^e_{t-1}].$$

Re-writing the above equation

$$\Delta\log P^e_t = w\ \Delta\log P_t + (1-w)\ \Delta\log P^e_{t-1} \tag{4}$$

With this theory of expectations, (4) can be substituted into (5) and a Koyck transformation performed with the result that

$$\begin{aligned}\Delta\log P_t &= \alpha w a_0 + (1 + \alpha a_3 = \alpha)\ \Delta\log P_{t-1} - \alpha a_3\ \Delta\log P_{t-2} \\ &+ \alpha a_1\ \Delta\log i_{t-1} + \alpha a_1 w \log i_{t-2} + \alpha(a_2-1)\ \Delta\log y_{t-1} \\ &+ \alpha a_2 w \log y_{t-2} + \alpha\ \Delta\log M_{t-1} - \alpha w \log V_{t-2}\end{aligned} \tag{8}$$

And if the redistribution term is omitted from (1) (i.e. $a_3 = 0$) we get

$$\begin{aligned}\Delta\log P_t &= \alpha w a_0 + (1-\alpha)\Delta\log P_{t-1} + \alpha a_1\ \Delta\log i_{t-1} \\ &+ \alpha a_1 w \log i_{t-2} + \alpha(a_2-1)\ \Delta\log y_{t-1} + \alpha a_2 w \log y_{t-2} \\ &+\alpha\ \Delta\log M_{t-1} - \alpha w \log V_{t-2}\end{aligned} \tag{8A}$$

Linear and non-linear regressions can be run on (8) and (8A).

EMPIRICAL RESULTS

Equations explaining the rate of inflation were estimated using seasonally-adjusted quarterly observations[20] for various periods, the longest being 1953:I to 1969:IV. Three alternative price indexes were used:

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[20]The three price indexes and any other variables which were not reported on a seasonally-adjusted basis were seasonally adjusted by the U.S. Census bureau's seasonal-adjustment package, X-11.

P_{IPI} - GNP implicit price deflator (1961 = 100)

P_{CPI} - Consumer price index (1961 = 100)

P_{WPI} - Wholesale price index (1935-39 = 100)

and three different interest rates were used:

i_{TB} - the yield on 91-day treasury bills

i_{0-3} - the yield on 0-3 year government of Canada bonds

i_{LT} - the yield on long-term government of Canada bonds.

Results are reported using the interest rate which yields the highest R^2. For the purpose of this section, the definition of the money supply that was used was:[21]

M_2 = currency outside chartered banks plus total adjusted Canadian dollar chartered bank deposits held by the general public.

The regression results for equations (5) and (5A) using the previously calculated expected rates of inflation are presented in Tables XI and XII respectively.[22] From Table XI a significant estimate of α is obtained only for the WPI. With the WPI index the estimate of α is 0.05. With all three price indexes positive income elasticities of velocity[23] are obtained. For the WPI a_2 is approximately 0.2[24] For the WPI, a_3 is positive (and equal to 3.7) but for the CPI and IPI a_3 appears to be negative. For all three price

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[21] In section four the definition of the money supply will be discussed.

[22] It should be noted that expectations of inflation were derived separately for each of the three price indexes using the Carr-Smith method.

[23] Exact standard errors cannot be derived for the income and interest elasticities of velocity.

[24] It should be noted that the income elasticity of the demand for real cash balances is $1 - a_2$. In this case it turns out to be 0.8.

indexes a perverse sign is obtained for the interest elasticity of velocity.[25] For the WPI a_1 is -0.4. The results in Table XII conform in general to the results in Table XI. The regression results for equation (5)´´ and (5A)´ are presented in Tables XIII and XIV respectively. For these results it was assumed that the weighting pattern of past rates of inflation in the formation of expectations of price inflation followed a second-degree polynomial function. With this assumption the Almon technique was employed with length of lags varying from 16 quarters to 32 quarters. The results for the length of lag which yielded the highest R^2 are reported in Tables XIII and XIV. In Table XIII, a statistically significant α is obtained for all three price indexes. For the WPI α is 0.05, for the CPI α is 0.05 and for the IPI α is 0.06. Income elasticities are positive for all three price indexes. The income elasticity of velocity (a_2) is 0.8 using the WPI, 0.3 using the CPI and 0.6 using the IPI. A positive interest elasticity of velocity (of 0.06) is obtained using the CPI. However the interest elasticity has the wrong sign in the results using the WPI and IPI. The results in Table XIV conform to the results in Table XIII but in Table XIV a positive interest elasticity is also obtained using the IPI.

Tables XV and XVII contain the ordinary least-squares estimates without applying any constraints for equations (8) and (8A) respectively. Equation (8) has 9 independent variables but only 6 parameters. Similarly (8A) has 8 independent variables but only 5 parameters. More efficient estimates than ordinary least-squares can be obtained for (8) and (8A) if account were taken of the non-linear constraints on the coefficients of these two equations. The results using a non-linear regression technique are presented in Tables XVI and XVIII. In Table XVI all the estimated para-

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[25] If interest is paid on bank deposits (as was the case in Canada for most of the sample period) then a_2 is biased downwards (since there is an omitted variable, bank interest, which should have a negative sign in (5) and (5A) and which is positively correlated to market interest rates). For an analysis of the effect of the payment of bank interest in the demand for money see B. Klein, "The Payment of Interest in Commercial Bank Deposits and the Price of Money: A Study of the Demand for Money", unpublished Ph.D dissertation, University of Chicago, 1970.

meters for all three price indexes have the correct sign. For the WPI statistically significant estimates are obtained for α, a_0 and a_2. However the estimate of 2.19 for a_2 is implausibly high. The implicit income elasticity of money demand turns out to be negative, -1.19. Since most demand for money studies have positive income elasticities one would expect a_2 to be less than one. Although a_2 is significantly different from zero at the 95 per cent significance level, it is not significantly different from one. For the CPI and IPI no estimates are statistically significant and in fact the R^2 for the IPI is negative. This would indicate that the model with adaptive expectations does not work well for the CPI and IPI. The estimate of w, using the WPI indicates rather large average lags[26] in the formation of expectations of inflation. For the WPI the average lag obtained is 51 quarters.[27] The result for the WPI indicates a rather small interest elasticity of velocity. Although the estimated interest elasticity is not significantly different from zero, if we increase our estimate by two standard deviations we get an estimate of the interest elasticity of 0.06. This result would indicate small interest rate effects for the demand for money. Also the redistribution term appears to have a positive effect on velocity. This means unexpected inflation redistributes income in favor of households with low average real cash balances (i.e. low k's).

Three sets of results have been presented above. Each set of results reflects a different method of estimating expectations of inflation.[28] These different methods were used since it was felt that the price expectations variable was a crucial variable for the assessment of the final results. Each set of results indicates that money influences the price level with a lag. The estimates of the lag vary somewhat with each set of results. Consider the results for the WPI. In Table XI using extraneous expectation data an α of 0.05 was obtained,

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[26] The average lag is $\frac{1-w}{w}$

[27] Although we do not have an exact standard error for the average lag, it can be seen that if the estimate of w is increased by two standard deviations the average lag will become 21 quarters. In general it is very difficult to get precise estimates of average lags.

[28] The fact that three alternative measures of expectations of inflation were used reflects a lack of knowledge by economists on the formation of expectations.

in Table XIII using Almon lags, an α of 0.05 was also obtained, and in Table XVI using adaptive expectations and a non-linear regression technique an α of 0.12 was obtained. The results suggest that α (which measures the lag in the effect of money in prices) is somewhere between 0.05 and 0.12.

Consider an α of 0.12. What does this mean? This means that if the model is in equilibrium and the rate of growth of the money supply is suddenly and unexpectedly increased, this causes actual velocity to fall and causes a gap between desired and actual velocity. Now an α of 0.12 means the rate of price change will increase by 12 per cent of the difference between desired and actual velocity within one quarter. Let us take an example to show exactly what this means for the speed of price adjustment. Suppose the economy has been in equilibrium for a long period of time. Suppose our economy is static with no growth in real income, with a constant money supply and with no inflation. Initially, suppose

$$M = 100 \qquad V = 4 \qquad p = 1.00 \qquad y - 400$$

and now suppose unexpectedly the money supply doubles to 200. Let us suppose everyone expects this to be a once-and-for-all increase in the money supply. Table XIX shows how prices adjust to this change assuming $\alpha = 0.12$. It can be seen that after one year about 32 per cent of the total required adjustment in prices is made. It takes about seven quarters for half of the adjustment in prices to take place. Table XIX clearly shows the lag in adjustment of prices to changes in the money supply. If α were equal to 0.05 it would take 19 quarters for half the adjustment in prices to take place. It appears that these lags are of considerable length and that we do not exactly know what they are. These lags should serve as a warning to monetary authorities not to expect instant results whenever they change monetary policy. The results here indicate a considerable lag in the transmission of changes in money to changes in the price level.

ALTERNATIVE DEFINITIONS OF THE MONEY SUPPLY

The subject of the appropriate definition of the money supply has been debated for a long time.[29] Many economists

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[29]For a discussion of this debate see M. Friedman and A. Schwartz, Monetary Statistics of the United States, New York 1970, chapters 1 and 2.

have tried to define money on a strictly theoretical basis. All of these attempts appear to be unsatisfactory. There is a whole continuum of financial assets and where we draw the line (on a theoretical basis) between some assets which we call money and other non-money assets would appear to be strictly an arbitrary decision. It would seem much more satisfactory if money was defined on an empirical basis. If money was defined on an empirical basis the definition of money might be different depending on what we are trying to explain. And this is as it should be. Also the definition of money would change as financial institutions changed and issued new types of liabilities.

In Canada there are a number of financial institutions which issue liabilities which appear to be very similar to the liabilities issued by chartered banks. Trust and mortgage loan companies offer deposits transferable by cheque. Credit Unions (in particular caisse populaires) offer deposits transferable by cheque. Government savings institutions offer pass-book savings. Which liabilities of which institutions should be included in the appropriate definition of the money supply? To answer this question the following eight monetary totals were calculated:

M_0 = currency outside banks plus chartered bank demand deposits held by the general public and adjusted for float

M_1 = currency outside banks plus total chartered bank chequable deposits held by the general public and adjusted for float

M_1 = currency outside banks plus bank demand deposits held by the general public and adjusted for float plus personal savings deposits

M_2 = currency outside banks plus total chartered banks deposits held by the general public and adjusted for float

M_3 = M_1 plus chequable deposits in trust companies plus chequable deposits in mortgage loan less cash reserves of trust and mortgage companies

M_4 = M_3 plus non-chequable deposits in trust and mortgage loan companies plus non-chequable and fixed term personal savings deposits at chartered banks

M_5 = M_2 plus total deposits at trust and mortgage loan less cash reserves of trust and mortgage companies

M_6 = M_5 plus total deposits at Quebec Savings Banks plus total deposits in credit unions plus total deposits in government savings institutions less cash reserves of Quebec Savings Banks and credit unions.

How does one choose among these alternative definitions? Here it is proposed to use that definition of the money supply which yields the best explanation of the rate of inflation. For purposes of the following experiment it was decided to use equation (8A) since this formulation was the only formulation which had the rate of growth of the money supply in it directly as an independent variable. Ordinary least-squares estimates of equation (8A) were obtained using P_{WPI} and i^{TB} and using our eight alternative definitions of the money supply. The R^2's and the standard error of estimate ($S_y.x$) for the eight equations are presented in Table XX. Also the rate of inflation was regressed against past rates of change in the money supply where

$$\Delta \log P_t = a + b \sum_{i=0}^{n} w_i \ \Delta \log M_{t-1} \qquad (14).$$

Equation (14) was estimated by the Almon technique assuming the weights followed a second-degree polynomial and assuming the length of the lag to be 16 quarters. The R^2's and standard errors of estimate for equation (14) for the eight definitions of money are also presented in Table XX.

From Table XX it appears that the broad definitions of money (M_2, M_5 and M_6) do better than the narrow definitions of money. It also appears that once you include all chartered bank deposits in the definition of money very little explanatory power is added by including liabilities of other financial institutions in the definition of money. In fact M_2 has the lowest Sy.x in the estimation of equation (8A). Therefore M_2 would seem to be the empirically appropriate definition of the money supply when looking at an explanation of inflation.

CONCLUSION

From the foregoing empirical results it can be concluded that: (1) The rate of growth of the money supply plays an important role in the determination of the rate of change of

the absolute price level. This result should not be too surprising. Since the absolute price level can be conceived of as the rate of exchange of money for goods (in general), the more money that is printed the less each unit of money is worth and the more of these units of money that have to be exchanged for a unit of goods (i.e. the higher the price level). Before the 1930s this result would have been expected by the majority of economists.

(2) Changes in the rate of growth of the money supply bring about changes in the rate of growth of the absolute price level but only after a considerable lag. The exact nature of the lag is a matter requiring further research.[30] For the WPI it can take anywhere from seven to 19 quarters for half of the effect of a change in the money supply to be reflected in prices. This result has important policy implications. When the Bank of Canada increases the rate of growth of the money supply and observes this has no immediate effect on the rate of inflation it should not falsely conclude that money has no influence on the level of prices. Conversely when the Bank of Canada decreases the rate of growth of the money supply in order to combat inflation and observes no immediate reduction in the rate of inflation it should <u>not</u> conclude one of the following:

(a) Monetary policy is ineffective in fighting inflation; therefore, other means such as direct price and wage controls must be used to combat inflation,

or

(b) The initial decrease in the rate of growth of money was too small and as a consequence the rate of growth of the money supply should be further reduced to have any significant effect in combatting inflation.

It is because of these lags in monetary policy that central banks tend to over-react to any given situation. When there is a need to follow an "easy" money policy they tend to expand the money supply too rapidly and as a consequence bring on inflation and when there is a need to fight inflation and so follow a "tight" money policy they tend to contract the money supply too much and bring on a recession.

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[30]In particular, work should be done where α is treated as a variable and not a parameter of the system.

(3) The empirical results suggest that the broad definition of money (M_2) which includes all publicly held chartered bank deposits works better in explaining the rate of inflation than narrower definitions of money. The results also suggest that there is not much to be gained in explaining the rate of inflation by including liabilities of financial intermediaries other than chartered banks in the definition of the money supply. For purposes of explaining the rate of inflation, M_2 appears to be the most appropriate definition of the money supply.

TABLE XI

Regression Results of Equation (5) Using Previously Calculated Expected Rate of Inflation

1958:III - 1969:IV+

Dependent Variable	Constant	$\log i^{TB}_{t-1}$	$\log i^{0-3}_{t-1}$	$\log i^{LT}_{t-1}$	$\log y_{t-1}$	$\log V_{t-1}$	$\Delta\log P_{t-1} - \Delta\log P^e_{t-2}$	R^2	Sy.x	D.W.
$\Delta\log P_t$	0.006			-0.021	0.012	-0.054	+0.20	0.20	0.0047	2.04
$- \Delta\log P^e_{t-1}$										2.16
using WPI	(0.08)			(-1.97)	(1.59)	(-2.29)	(1.42)	Implied R^2 on $\Delta\log P_t$ = 0.26		
$\Delta\log P_t$										
$- \Delta\log P^e_{t-1}$	-0.25			-0.019	0.026	(0.004)	-0.23	0.46	0.0028	1.80
using CPI	(-5.43)			(-3.02)	(4.93)	(0.26)	(-1.71)	Implied R^2 on $\Delta\log P_t$ = 0.62		
$\Delta\log P_t$ $- \Delta\log P^e_{t-1}$	-0.27			-0.021	0.029	-0.0017	-0.37	0.31	0.0049	1.90
using IPI	(-3.37)			(-1.96)	(3.19)	(0.06)	(-2.69)	Implied R^2 on $\Delta\log P_t$ = 0.43		

+ Bracketed values are the t-statistics. R^2 is the coefficient of determination, Sy.x is the standard error of estimate and D.W. is the Durbin-Watson statistic.

TABLE XII

Regression Results of Equation (5A) Using Previously Calculated Expected Rate of Inflation, 1958:III - 1969:IV

Dependent Variable	Constant	$\log i^{TB}_{t-1}$	$\log i^{0-3}_{t-1}$	$\log i^{LT}_{t-1}$	$\log y_{t-1}$	$\log V_{t-1}$	R^2	Sy.x	D.W.
$\Delta \log P_t - \Delta \log P^e_{t-1}$	0.0068			-0.022	0.013	-0.060	0.16	0.0047	1.60
using WPI	(0.096)			(-2.16)	(1.78)	(-2.66)	Implied R^2 on $\Delta \log P_t$ = 0.22		
$\Delta \log P_t - \Delta \log P^e_{t-1}$	-0.21			-0.021	0.024	-0.004	0.36	0.0032	2.23
using CPI	(-4.89)			(-3.20)	(4.49)	(-0.20)	Implied R^2 on $\Delta \log P_t$ = 0.50		
$\Delta \log P_t - \Delta \log P^e_{t-1}$	-0.22			-0.022	0.025	-0.018	0.16	0.0055	2.61
using IPI	(-2.79)			(-1.90)	(2.58)	(-0.60)	Implied R^2 on $\Delta \log P_t$ = 0.24		

TABLE XIII

Regression Results of Equation (5)" Using Second-Degree Almon Variables,

1955:III - 1969:IV

Dependent Variable	Constant	log i^{TB}_{t-1}	log i^{0-3}_{t-1}	log i^{LT}_{t-1}	log y_{t-1}	Δlog P_{t-1}	log V_{t-1}	Z^*_1	Z^*_2	Length of Lag	R^2	Sy.x	D.W
Δlog P_t	-0.31			0.0088	0.041	-0.021	-0.054	-3.68	0.50	24	0.42	0.0043	2.0
using WPI	(-2.80)			(-1.24)	(3.89)	(-0.17)	(-2.92)	(-1.93)	(0.49)				
Δlog P_t	-0.08	0.0027			0.012	0.045	-0.046	-4.02	0.21	20	0.46	0.0033	1.9
using CPI	(-1.94)	(1.45)			(2.83)	(0.34)	(-2.60)	(-3.24)	(3.10)				
Δlog P_t	-0.27			-0.019	0.035	-0.289	-0.063	-1.76	1.77	16	0.35	0.0054	2.4
using IPI	(-2.71)			(-2.00)	(3.50)	(-2.32)	(-2.05)	-0.67	(0.74)				

* Z_1 and Z_2 are respectively first and second degree Almon variables created on the appropriate rate of inflation for each regression.

TABLE XIV

Regression Results of Equation (5A)´ Using Second-Degree Almon Variables

1955:III - 1969:IV

Dependent Variable	Constant	$\log i_{t-1}^{TB}$	$\log i_{t-1}^{0-3}$	$\log i_{t-1}^{LT}$	$\log y_{t-1}$	$\log V_{t-1}$	Z_1^*	Z_2^*	Length of Lag	R^2	Sy.x	D.W.
$\Delta \log P_t$	-0.28	-0.0035			0.039	-0.061	-5.17	1.86	24	0.39	0.0043	1.68
using WPI	(-2.73)	(-1.44)			(4.00)	(-3.68)	(-3.02)	(1.80)				
$\Delta \log P_t$	-0.07	0.0020			0.011	-0.038	-3.63	3.19	20	0.46	0.0033	2.09
using CPI	(-1.76)	(1.18)			(2.59)	(-2.34)	(-3.23)	(3.30)				
$\Delta \log P_t$	-0.24	0.0049			0.024	-0.013	5.02	-5.25	32	0.26	0.0060	2.23
using IPI	(-2.36)	(1.55)			(2.72)	(-0.43)	(1.92)	(-1.97)				

*Z_1 and Z_2 are respectively first and second degree Almon variables created on the appropriate rate of inflation for each regression.

TABLE XV

Regression Results of Equation (8), Without Constraints,

1953:I - 1969:IV

Dependent Variable	Constant	$\Delta\log P_{t-1}$	$\Delta\log P_{t-2}$	$\Delta\log i^{TB}_{t-1}$	$\log i^{TB}_{t-2}$	$\Delta\log Y_{t-1}$	$\log Y_{t-2}$	$\Delta\log M_{t-1}$	$\log V_{t-2}$	R^2	Sy.x	D.W.
$\Delta\log P_t$	0.040	0.156	0.099	0.001	0.001	0.132	0.0008	0.129	-0.028	0.50	0.0042	2.11
using WPI	(0.60)	(1.39)	0.90	(0.34)	(0.68)	(2.99)	0.138	(2.73)	(-1.96)			
$\Delta\log P_t$	-0.02	0.124	0.039	0.003	0.004	-0.007	0.003	0.046	-0.012	0.45	0.0035	2.02
using CPI	(-0.46)	(0.95)	(0.31)	(1.07)	(1.69)	(-0.20)	(0.64)	(1.17)	(0.84)			
$\Delta\log P_t$	-0.12	-0.282	-0.210	0.009	0.003	0.007	0.016	0.027	-0.027	0.31	0.0062	2.04
using IPI	(-1.46)	(-2.23)	(-1.75)	(1.74)	(0.81)	(0.12)	(1.78)	(0.37)	(-1.49)			

TABLE XVI

Regression Results of Equation (8), With Appropriate Constraints Applied,

1953:I - 1969:IV

Dependent Variable	α	w	a_o	a_1	a_2	a_3	R^2	Sy.x	D.W.
$\Delta\log P_t$ using WPI	0.120 (2.69)	0.018 (1.30)	-21.86 (-3.14)	0.0014 (0.05)	2.19 (3.32)	2.32 (1.81)	0.29	0.0050	2.76
$\Delta\log P_t$ using CPI	0.041 (1.36)	0.358 (0.873)	-4.20 (-0.81)	0.062 (0.82)	0.505 (1.0)	8.35 (1.19)	0.24	0.0040	2.75
$\Delta\log P_t$ using IPI	0.093 (1.33)	0.068 (1.50)	-19.87 (-1.45)	0.107 (1.13)	2.03 (1.53)	6.55 (1.14)	-0.26	0.0081	2.94

TABLE XVII

Regression Results of Equation (8A), Without Constraints,

1953:I - 1969:IV

Dependent Variable	Constant	$\Delta\log P_{t-1}$	$\Delta\log i^{TB}_{t-1}$	$\log i^{TB}_{t-2}$	$\Delta\log y_{t-1}$	$\log y_{t-2}$	$\Delta\log M_{t-1}$	$\log V_{t-2}$	R^2	Sy.x	D.W.
$\Delta\log P_t$	0.035	0.18	0.0015	0.0021	0.138	0.0015	0.124	-0.029	0.50	0.0042	2.16
using WPI	(0.52)	(1.70)	(0.45)	(0.77)	(3.18)	(0.26)	(2.64)	(-2.05)			
$\Delta\log P_t$	-0.025	0.13	0.0029	0.0040	-0.0090	0.0034	0.047	-0.012	0.45	0.0035	2.01
using CPI	(0.52)	(1.02)	(1.06)	(1.76)	(-0.25)	(0.72)	(1.20)	(-0.87)			
$\Delta\log P_t$	-0.086	-0.23	0.0093	0.0035	0.002	0.012	0.058	-0.034	0.27	0.0063	2.23
using IPI	(-1.01)	(-1.81)	(1.83)	(0.85)	(0.004)	(1.32)	(0.83)	(-1.34)			

TABLE XVIII

Regression Results of Equation (8A), With Appropriate Constraints Applied

1953:I - 1969:IV

Dependent Variable	α	w	a_o	a_1	a_2	R^2	Sy.x	D.W.
$\Delta\log P_r$	0.18	-0.0092	-13.35	-0.012	1.63	0.15	0.0054	2.70
using WPI	(4.0)	(-0.95)	(-3.49)	(-0.55)	(4.66)			
$\Delta\log P_t$	0.082	-0.011	-8.07	0.030	0.937	0.08	0.0044	2.80
using CPI	(2.28)	(-0.37)	(-1.53)	(0.78)	(1.79)			
$\Delta\log P_t$	0.244	-0.0050	-11.21	0.044	1.42	-0.65	0.0092	2.98
using IPI	(3.42)	(-0.38)	(-1.35)	(1.56)	(3.36)			

TABLE XIX

Effect On Prices Of A Once-And-For-All Doubling Of The Money Supply Assuming $\alpha = 0.12$

Quarter	M	V	P	y	$\log V^d$	$\log V$	$\log V^d - \log V$	$\Delta \log p \times 100$
0	100	4	1.00	400	1.386	1.386	0	0
1	200	2	1.00	400	1.386	0.693	0.693	0
2	200	2.173	1.087	400	1.386	0.776	0.610	8.3
3	200	2.338	1.169	400	1.386	0.849	0.537	7.3
4	200	2.494	1.247	400	1.386	0.914	0.472	6.4
5	200	2.639	1.320	400	1.386	0.971	0.415	5.7
6	200	2.774	1.387	400	1.386	1.020	0.366	5.0
7	200	2.899	1.449	400	1.386	1.064	0.322	4.4
8	200	3.012	1.506	400	1.386	1.102	0.283	3.9
∞	200	4	2.00	400	1.386	1.386	0	0

TABLE XX

Comparison Of The R^2 And Sy.x Of Equation (8A) And Equation (12) Using The WPI To Define The Rate Of Inflation And Using 8 Alternative Definitions Of The Money Supply, 1953:I - 1969:IV

Definition of the Money Supply	Equation (8A)		Equation (14)	
	R^2	Sy.x	R^2	Sy.x
M_0	0.44	0.00449	0.23	0.00488
M_1	0.41	0.00458	0.10	0.00528
M_1'	0.45	0.00444	0.28	0.00471
M_2	0.50	0.00424	0.34	0.00451
M_3	0.41	0.00458	0.10	0.00528
M_4	0.45	0.00444	0.30	0.00466
M_5	0.49	0.00427	0.35	0.00447
M_6*	0.44	0.00428	0.29	0.00440

* Was run for the period 1953:I - 1967:IV. R^2 for M_6 should not be compared to other R^2's.

THE EMPIRICAL DEFINITION OF MONEY

by

John W.L. Winder

In his brilliant survey article of one decade ago, Harry Johnson assessed what is perhaps the central problem in monetary economics as follows:

> "...the chief substantive issues outstanding are these: first, what specific collection of assets corresponds most closely to the theoretical concept of money--an issue that arises as soon as the distinguishing characteristic of money ceases to be its function as a medium of exchange; second, what the variables are on which the demand for money so defined depends; and third, whether the demand for money is sufficiently stable to provide, in conjunction with the quantity of money, a better explanation of observed movements of money income and other aggregates than is provided by models built around income-expenditure relationships. These are essentially empirical issues, to which empirical research has as yet produced no conclusive answers; and they clearly have an important practical bearing on monetary policy."[1]

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[1] Johnson, H.G., "Monetary Theory and Policy," American Economic Review, June 1962, pp. 344-5.

The ultimate relevance of empirical studies of the demand for money bears on the fundamental issue as to "whether monetary theory is more usefully formulated in terms of the demand for and supply of money or of the influence of money on expenditure and income".[2] The appropriate formulation of monetary policy is of course contingent upon the resolution of this issue, and embedded within it is the question of the relevant concept of money. Effective monetary policy requires an empirically established definition of money.

Empirical definitions, however, will not be unique. They need not remain the same over time, and may differ according to differences in the questions which alternative models may be designed to answer. For example, reduced form models may be constructed to assess the role of money in price and income determination. Viewed by the precepts of such models, some definitions of money may be found to perform better than others - to be empirically relevant. Similarly, given specifications of the aggregate demand function for money, by their relative performance under alternative definitions of money imply that definition which is empirically most appropriate, at least for the given specification.

In a recent contribution of his own toward an empirical definition of money, George Kaufman cited what he considered to be the three most promising techniques among those already applied to U.S. data.[3] These were: (1) definition by cross-elasticity of substitution of financial assets; (2) definition by discriminant analysis of time series characteristics, and (3) definition by ability to explain statistically aggregate nominal income. Kaufman's own work related to the third approach, employed by Milton Friedman and David Meiselman in their study for the Commission on Money and Credit. Evaluation of the relative merits of alternative techniques would constitute a substantial and undoubtedly contentious undertaking in its own right. The objective of the present paper is the more modest one of approaching the empirical definition of money through estimation of a demand function of specified form.

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[2] Ibid., p. 356

[3] Kaufman, George C., "More On An Empirical Definition of Money", American Economic Review, March 1969, pp. 78-87.

The demand for money has been specified and estimated in a wide variety of functional forms and with a variety of determining variables. In his excellent book on the subject, David Laidler states that most recent empirical work on the matter has postulated that the demand for money function may be approximated by:

$$\ln(M/P)_t = \ln k + a \ln(X)_t - b \ln r_t \qquad (1)$$

where X may stand for the level of income, non-human wealth, or permanent income.[4] In this paper, the level of current real income is used: $X = (Y/P)$. The other symbols as used here relate to current nominal gross national product (Y), its implicit deflator (P), the nominal money stock (M), and the treasury bill rate (r).

In his book, Laidler goes on to point out that tests of this sort ignore the "identification problem" and therefore in principle require supplementation by work such as that done by Brunner and Meltzer,[5] and Teigen[6] who simultaneously fitted supply and demand functions for money. The results obtained were essentially the same as those obtained with similar studies which did not take account of the identification problem. Laidler concludes:

> "In short then, whether one thinks of the demand for money function as being constrained by income, wealth, or expected income, whether one chooses to define money to include time deposits or exclude them, whether one chooses to ignore the identification problem or deal with it, whether one uses a short rate of interest, a long one, the return on financial intermediaries' liabilities or the yield on corporate equities, there is an overwhelming body of evidence in favor of the proposition that the demand for money is stably and negatively related to the rate of

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[4] Laidler, D.E.W., The Demand for Money: Theories and Evidence, Scranton, 1969.

[5] Brunner, K., and Meltzer, A.H., "Some Further Evidence on Supply and Demand Functions for Money", Journal of Finance, May 1964, pp. 240-283.

[6] Teigen, R., "Demand and Supply Functions for Money in the United States", Econometrica, October 1964, pp. 477-509.

interest. Of all the issues in monetary economics, this is the one that appears to have been settled most decisively."[7]

No formal attempt to take account of the identification problem is made in the work that follows. This is in part because of the empirical evidence just cited to the effect that it is not of critical significance so far as the demand for money is concerned, in part because of evidence to the effect that the money supply in Canada is substantially determined in passive response to a predetermined interest rate target or state of general credit conditions,[8] and in part because of the need to limit the scope of this particular investigation.

The choice of the functional form indicated in equation (1) was also based on some experience derived in the course of investigations into the role of price and interest expectations in the demand for money in Canada with Professor L.B. Smith.[9] In the development of that research various functional forms appearing in the literature were employed but the form reported upon in the paper and indicated above seemed to yield greater stability of results and permit most reasonable interpretation. In the study, results were reported for three definitions of the interest rate and two definitions of money. In addition to the yield on 91-day treasury bills the interest rates were the yield on short-term (0-3 year) Government of Canada bonds and the yield on long-term Government of Canada bonds. Since the results did not differ in any substantive way among these alternatives the treasury bill rate only was used throughout the investigations undertaken here. In the cited study, money was defined as the sum of currency outside the banking system

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[7] Laidler, op. cit., p. 97.

[8] Support of this position may be found in the following: Courchene, T.J., "Recent Canadian Monetary Policy", Journal of Money, Credit and Banking, February 1971, pp. 35-56; and Helliwell, J.F., et. al., The Structure of RDX2, Bank of Canada Staff Research Studies, 1971.

[9] Smith, L.B., and Winder, J.W.L., "Price and Interest Rate Expectations and the Demand for Money in Canada", Journal of Finance, June 1971, pp. 671-82.

and demand deposits in the chartered banks, with and without notice deposits in the chartered banks. In the present investigation these components were handled separately and other alternatives were also considered, as indicated below.

The model of expectations employed in the earlier work has also been used here. It is based upon the synthesis by Modigliani and Sutch of normative and extrapolative models of expectations.[10] As applied to interest rates, the normal level may be denoted by $\bar{r}$ and the expected level by r^*. The normal level is expressed by the relation,

$$\ln \bar{r}_t = v \sum_{i=1}^{m} u_i \ln r_{t-1}$$

$$+ (1 - v) \ln z; \; 0 < v < 1; \; \sum_{i=1}^{m} u_i + 1 \qquad (2)$$

where z is a constant representing a very long-run normal level, and v and $(1 - v)$ are constants to provide a weighted average of z with the expression $\sum_{i=1}^{m} u_i \ln r_{t-i}$ which is itself a weighted average of recent past rates.

If the interest rate expectations are normative or regressive, the level of interest rates is expected to fall when the current level exceeds the normal level. This hypothesis may be formulated as:

$$\ln(r^*_t/r_t) = \alpha_1 \ln(\bar{r}_t/r_t) = \alpha_1(1-v) \ln z$$

$$+ \alpha_1 v \sum_{i=1}^{m} u_i \ln r_{t-i} - \alpha_1 \ln r_t \qquad (3)$$

where α_1 is the elasticity of normative expectational responsiveness.

If interest rate expectations are extrapolative only the recent past is important, so the very long-run level, z, does not enter. The level of interest rates is expected to

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[10] Modigliani, F., and Sutch, R., "Innovations in Interest Rate Policy", American Economic Review, May 1966, pp. 178-197.

rise when the current level exceeds the average level of the recent past. In other words, the direction of recent past changes in interest rates is expected to continue into the future. This hypothesis may be expressed as:

$$\ln(r_t^*/r_t) = \alpha_2(\ln r_t - \sum_{i-1}^{n} \gamma_i \ln r_{t-i});\ \alpha_2>0,$$

$$\sum_{i=1}^{n} \gamma_i = 1,\ n < m \qquad (4).$$

Because the range of relevant past experience should be shorter for extrapolative than for normative expectations, n should be considerably smaller than m. The elasticity of extrapolative expectational responsiveness is α_2.

Expectations may, of course, be based upon both normative and extrapolative elements. Combining (3) and (4) yields the expression:

$$\ln(r_t^*/r_t) = \alpha_1(1-v)\ln z + (\alpha_2 - \alpha_1)\ln r_t$$

$$+ \sum_{i=1}^{m} (\alpha_1 v u_i - \alpha_2\gamma_i)\ln r_{t-i} \qquad (5).$$

The shape of the net lag structure can only be determined empirically. It will be the result of the relative strengths of the extrapolative and normative elements embodied in the terms $(\alpha_1 v u_i - \alpha_2\gamma_i)$. The extrapolative element $\alpha_2\gamma_i$, because of the shorter horizon, will exert its strongest influence for the most recent past periods, with the normative element $\alpha_1 v u_i$ dominant farther back in time. Inclusion of hypothesis (5) in the demand function of form (1) yields:

$$\ln(M/P)_t = [\ln k + c\alpha_1(1 - v) \ln z] + a \ln(Y/P)_t$$

$$- [b + c(\alpha_1 - \alpha_2)]\ln r_t + c \sum_{i=1}^{m} (\alpha_1 v u_i - \alpha_2\gamma_i)\ln r_{t-i} \qquad (6)$$

where c is the elasticity of demand for money with respect to an expected change in the rate of interest as measured by r_t^*/r_t.

In the tables which follow results are presented for the basic specification of the demand for money (including only real income and the treasury bill rate) as well as extensions

of it to incorporate interest and/or price expectations. For each alternative specification of independent variables the following alternative possible measures of money have been used to construct dependent variables:

C	Currency in hands of public (excluding coin).
D	Demand deposits at chartered banks (excluding float).
PN	Personal savings deposits at chartered banks.
ON	Other term and notice deposits at chartered banks.
TD	Demand and saving deposits at trust companies.
GIC	Term deposits and guaranteed investment certificates at trust companies.
MD	Demand and savings deposits at mortgage loan companies.
MT	Certificates, debentures and term deposits at mortgage loan companies.
M1	C + D
M2	C + D + PN +
M3	C + D + PN + ON

All data were quarterly and not adjusted for seasonality. Dollar magnitudes were in billions and this is signified in the tables by the number 9 in a variable symbol. The prefix L indicates natural logs and the suffix R deflation by the price index. For example,

LY9R signifies the natural log of
Y9R = Y9/P = GNE in billions of 1957 dollars, where
P = GNE price deflator (1957 = 1.00)

In addition, the interest rate used was:

TBR three-month treasury bill rate, average of weekly tenders, last-month-of-quarter.

The results for the basic specification appear in Table XXI, estimated for the period 1955:I through 1968:IV. The reported coefficients for real income and bill rate are of course in the form of elasticities; their t-statistics appear in parentheses below. The values of the coefficient of determination $\bar{R}^2$, are adjusted for degrees of freedom, but are not net of seasonality. The values of the Durbin-Watson statistic, DW, are presented in the final column.

It is evident that the basic specification explains most of the variation in each of the categories. Given the trend in real income and each of the dependent variables, this is

not at all surprising. The fundamental proposition to the effect that the demand for money is interest-elastic is confirmed for all categories, except C and PN, in spite of the fact that own rates have not been brought into the specification. The income-elasticities are much lower for C, D and PN than for the other components. These differences in interest and income elasticities are, of course, reflected in the aggregative relationships for M1, M2, and M3.

The values of the Durbin-Watson statistic indicate severe serial correlation. Much of the empirical literature on the demand for money reveals this to be characteristic of results, more so for quarterly than for annual data. Many other studies simply fail to report these statistics. When seasonally-adjusted data are used, some of the serial correlation may arise from the seasonal-adjustment process.

There are essentially three approaches to consider when confronted with serial correlation. In the first place, an auto-regressive transformation of the data could be called for. There is again evidence in the literature to the effect that such transformations have little impact on the results. Of most immediate relevance, this is what turned out to be the case in the Smith-Winder study cited earlier. Alternatively, it seems plausible to believe that adjustment lags may be important, particularly when quarterly data are employed, and therefore that some sort of adjustment model should be applied. The trouble with this approach is that it is fundamentally indistinguishable from a specification which calls for permanent or expected values of variables rather than current values in the function.[11] It is, of course, equally the trouble with trying to introduce interest rate expectations through inclusion of lagged values as done in this paper.[12] The final basic approach to the problem of serial correlation is to look for one or more omitted relevant explanatory variables. To the extent that the omitted variables are expected rather than measured values,

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11 Laidler, op. cit., p. 101.

12 The ambiguity in this approach is stressed explicitly by Professor Clinton in his 'Comment' on the Smith-Winder results, Journal of Finance, forthcoming. Professor Clinton also urges incorporation of income lags (accepting the ambiguity of their interpretation).

this approach is indistinguishable from the second, the allowance for adjustment lag. The present study omits "own rates" of return for all categories which have them. In part this is because the objective was to conduct a preliminary scan of potential candidates for inclusion under the banner of "money" within the context of a plausible specification of a basic yet generally acceptable form, rather than to finally determine the precise and statistically inviolate form of the demand for money. In part it is because experimentation with "own rates" seemed unpromising, at least within the context of the specification employed here.

In elaboration of this latter point, it may be noted that an examination of the residuals from the estimated equations of Table XXI(a) indicates that the search for an omitted variable may not be an easy one. With one or two exceptions, it is not at all clear that there is much in common among the "omitted" variables. In illustration of this observation, consider the following correlations:

Correlation Matrix of Residuals in Table XXI(a)

	C	D	PN	ON	TD	GIC	MD
D	0.44						
PN	0.81	0.44					
ON	0.39	0.54	0.47				
TD	-0.14	0.18	0.04	0.09			
GIC	0.25	0.08	0.39	0.05	0.66		
MD	-0.19	0.46	-0.01	0.51	0.70	0.12	
MT	0.15	0.03	0.27	-0.01	0.78	0.95	0.23

Errors seem for the most part to be unassociated. The noteworthy exceptions, in order of magnitude, are those for GIC and MT, C and PN, TD and MT, TD and MD, and TD and GIC. There is little encouragement here to search for a common

explanation. For example, it would be plausible *a priori* to suppose that the neglected variables relate to substitutability and are therefore probably rate differentials among categories. No doubt there is validity in this view but these residual patterns do not point that way at all. There is, for example, almost no association between residuals from equations relating to bank deposit liabilities on the one hand and those of the non-banks on the other. Moreover, on grounds of omitted allowance for substitutability variables one would expect negative association between residuals - there are only four such categories and they are too insignificant to matter.

One, admittedly arbitrary, way to allow for stock adjustment is to add to each regression a lagged value of the dependent variable. This has been done in Table XXI(b). The Durbin-Watson statistics now reflect their bias towards 2. The long-run or equilibrium elasticities with respect to real income and the treasury bill rate for the various categories are as follows:

		Y9R	TBR
1)	C9R	0.45	-0.02
2)	D9R	0.80	-0.20
3)	PN9R	1.44	-0.33
4)	ON9R	5.00	-0.69
5)	TD9R	7.00	-4.50
6)	GIC9R	4.67	-0.89
7)	MD9R	4.22	-1.22
8)	MT9R	2.00	-2.00
9)	M19R	0.71	-1.43
10)	M29R	0.95	-0.19
11)	M39R	1.54	-0.31

The long-run income-elasticities imply the same distinction between the deposit-instruments of the chartered banks and the non-banks observed earlier and the long-run interest-elasticities seem to reinforce this.

As an alternative, an autoregressive transformation was applied to the basic specification of Table XXI. These results appear in Table XXI(c). Except in the cases of C, D, and PN (and therefore M1 and M2) the sum of squared residuals decreases monotonically as RHO approaches unity. In all other categories, therefore, the best value of RHO implies taking first differences. The corresponding first difference equations naturally yield elasticities (and t-statistics) with respect to Y9R and TBR almost identical to those reported in Table XXI(c). The $\overline{R}^2$ for a first-difference equation is inevitably lower, of course, and very low in a couple of cases. These latter values are reported in the final column as "Net" $\overline{R}^2$ (although they are still not net of seasonality). In the cases of C, D, PN, M1, and M2 these values refer not to first-difference equations (RHO = 1), but to equations transformed by the corresponding values of RHO in the preceding column.

At face value the procedure is devastating in its impact on the earlier results. In addition to the one or two cases in which the quarter-to-quarter fit is negligible, there is no support for the earlier result to the effect that income-elasticities are substantially higher for non-bank deposit instruments, and there is virtual elimination of any consequential or significant interest rate elasticity. However, these results should be interpreted with caution; they are quite sensitive to small changes in RHO when RHO is in the neighbourhood of 1 - and yet small changes in RHO towards 1, while reducing the sum of squared residuals, may not do so "all that much".

Table XXI(d) illustrates this point. The equations in Table XXI(d) have been transformed not by the "optimal" value of RHO but by the autoregressive coefficient of the residuals in the corresponding equation of Table XXI(a), that is to say by setting RHO = b in an estimated equation of the form,

$$RESID_t = a + bRESID_{(t-1)}$$

for each category. According to this procedure, the distinc-

tion among income elasticities remains, but the interest elasticities are still much lower than in the untransformed equations.

It is doubtful that any such transformation represents the best way to approach the problem of serial correlation. In the cases which yield values of RHO near unity it may be that serial correlation is of a higher order. In any event, the preferred approach would be to enrich the specification, either by allowance for lagged adjustment (although the mere inclusion of lagged dependent variables is perhaps not the best way to go about it), or the incorporation of additional explanatory variables, or both. Although what is to follow is in principle a step in the right direction, it will be seen that it does not free us from the problem.

When the basic specification is expanded to allow for interest rate expectations the results are as reported in Table XXII. The net lag structure of the postulated interest expectational process has been estimated by a second degree polynomial constrained to go through zero in the twelfth lagged quarter.[13] (Since the current quarter value of LTBR is included separately, 12 quarterly values are included in all.) For economy in presentation, constants, coefficients on seasonal dummies, and coefficients on the Almon variables are not reported. None of these are of interest in themselves and the latter vary from one computational program to another. The disentangled lagged weights are program-invariant,however, and also constitute the interesting result, so these are reported. The values in parentheses beneath these estimates are <u>standard errors</u>.

The distinction among income elasticities remains in Table XXII, although the values differ somewhat from their counterparts in Table XXI. The positive signs for the current value of the treasury bill rate in the cases of C and PN suggest that the influence of extrapolative expectations dominates the opportunity cost element, which may have a very small influence in these cases. The inclusion of PN within M2 and M3 contributes toward the offsetting of these two effects in these aggregates. The lagged coefficients on the treasury

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[13] Almon, S., "The Distributed Lag Between Capital Appropriations and Expenditures", <u>Econometrica</u>, January 1965, pp. 178-196.

bill rate begin negative and revert to positive in all cases, which is consistent with the basic expectational hypothesis which posits a dominance of extrapolative expectations for the recent past. The degree and duration of such dominance varies from one category to another, however, extending over five or six periods in the cases of D, ON, and MD, about half that for TD and fewer still for the rest of the basic components. It is of course not possible to say whether these differences in timing among categories are statistically significant. Nevertheless the results invite such comparison and imply some conformity among all demand deposits on the one hand and all notice deposits on the other. On this view, C and ON seem to be out of line.

The results in Table XXIII reflect a further modification of the basic specification incorporating a second degree polynomial distributed lag on real income, beginning with the current quarter and constrained to pass through zero in the twelfth lagged quarter. The estimated coefficients may be interpreted as reflecting a combination of lagged adjustment and something approximating "permanent" income. As pointed out earlier, similar ambiguity prevails in the interpretation of the interest rate coefficients (and this is, of course, true of the results in Table XXII as well). The precise interpretation placed upon the results is necessarily a matter of judgment. The income coefficients and their standard errors are presented in the last two rows for each category.

The impact of inclusion of lagged values of income differs between currency and the deposit liabilities of chartered banks on the one hand and the deposit liabilities of the trust and mortgage loan companies on the other. In the former case the coefficients on lagged rates no longer become significantly positive; indeed for ON they remain significantly negative throughout. It is possible to view this result as the combined effect of lagged adjustment to opportunity cost and extrapolative interest expectations with the relative role of each a matter of judgment. The significantly positive coefficient for the current value of the treasury bill rate in the equations for C and PN implies some role for extrapolative expectations, however.

In the case of the deposit liabilities of trust and mortgage loan companies, the pattern of weights on lagged rates is very strongly that expected under the combined extra-

polative - normative hypothesis - although it is equally impossible to separately identify the impact of lagged adjustment. Compared with the other categories, however, adjustment lags and the duration of extrapolative dominance would appear to be shorter.

The difference in rate patterns for the trust and mortgage loan companies is accompanied by a difference in the pattern of coefficients on lagged income. These all become negative after five quarters or so. These negative values are not consistent with lagged adjustment or permanent income. It could be argued that the appropriate budget constraint is wealth, but permanent income, even as approximated by the present technique, should be a better proxy for wealth than these results would suggest. Whatever the reasons, the results of this specification establish a vivid empirical line between money as more or less narrowly defined and some broader alternative definitions.

A further generalization of the basic approach to accommodate a possible role of inflationary expectations is reported upon in Table XXIV. The quarter-to-quarter proportional rate of change in the implicit deflator for GNP was first regressed on seasonal dummies and then adjusted for the implied seasonality. Because negative values remained even after such adjustment, the log transformation was not applied to this series (referred to in the table as SAPDOT). The distributed lag was again estimated by a second degree polynomial beginning with the current quarter and constrained to go through zero in the twelfth lagged quarter. The estimated coefficients and their standard errors appear as the last two rows of each category.

Inflationary expectations are likely to be purely extrapolative, leading to a reduction in the demand for real balances. The preponderance of significantly negative coefficients is consistent with this hypothesis. There is again some distinction to be made between the trust and mortgage loan company deposits and the other categories. In the case of these deposits, the coefficients are significantly negative throughout. For notice deposits of the banks such significance disappears after six or seven lagged quarters. For bank demand deposits and currency it disappears after four lagged quarters. In the case of currency, significantly positive values appear from the eighth lagged quarter on.

Unfortunately, it is probably not possible to interpret the price coefficients unambiguously either. No doubt they do incorporate the impact of inflationary expectations. But given the specification in terms of real income and real balances they may also reflect lagged adjustment to the extent that realized price increases erode real balances, thereby necessitating a subsequent realignment with the higher price level. This may be the reason for the significantly positive signs for currency. The greater the impact of inflationary expectations the less tendency would there be for such realignment to take place. Such impact therefore seems less prominent for bank deposits than for non-bank deposits, and less prominent still for currency.

Introduction of price expectations does not materially alter the interest rate and income patterns established in Table XXIII except in the cases of term deposits of the trust and mortgage loan companies, for which recent interest coefficients are no longer significantly negative, and demand deposits of mortgage loan companies for which remote interest coefficients are no longer significantly positive. In the latter case also, the negative lagged income coefficients are either insignificant or borderline. It therefore turns out that demand deposits of mortgage loan companies now bear more resemblance to bank deposits than to other non-bank deposits, so far as interest rate and income are concerned, but not in relation to price change. Trust and mortgage loan company term deposits seem dominated by normative rather than extrapolative rate expectations or perhaps reflect a failure to distinguish between real and nominal interest rates.

In summary, the various components for possible inclusion within the definition of money differ more in degree than in substance in response to the explanatory variables which have been introduced in the specifications investigated here. The relatively clear boundary line between currency and bank deposits on the one hand and non-bank deposits on the other, which emerged from the basic specification, can be made to disappear by a first-difference transformation of the data since income elasticities then do not differ. Alternatively, it can be blurred by inclusion of lagged interest rates alone, which leaves intact the income-elasticity differentiation but tends to align demand deposits in one group and term deposits in another so far as rate patterns go. The boundary line is then re-established by the incorporation of income

lags as well as rate lags, only to be blurred again by the further inclusion of current and lagged rates of change of prices, the impact of which, so far as interest and income lag patterns go, is to lump everything together except for term deposits of trust and mortgage loan companies. But all deposits of these companies reflect a greater impact of inflationary expectations than do currency and bank deposits, and to this extent the original line remains. To the extent that substitutability between bank and non-bank deposits inhibits anti-inflationary monetary policy, the differentially adverse impact of inflation on the non-banks represents something of an automatic stabilizer.

TABLE XXI(a)

BASIC SPECIFICATION: 1955:I - 1968:IV

		Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW
1)	LC9R	-0.44 (-12.25)	-0.01 (-1.79)	-0.03 (-4.43)	-0.05 (-7.36)	0.42 (21.46)	0.003 (0.35)	0.95	0.65
2)	LD9R	-0.05 (-0.73)	-0.01 (-0.66)	-0.04 (-2.91)	-0.08 (-5.94)	0.66 (18.34)	-0.107 (-5.96)	0.90	0.70
3)	LPN9R	0.21 (3.07)	0.09 (6.34)	0.03 (2.20)	-0.04 (-2.73)	0.78 (20.75)	0.007 (0.36)	0.94	0.36
4)	LON9R	-8.28 (-24.76)	0.39 (5.80)	0.09 (1.38)	-0.36 (-5.15)	3.84 (21.39)	-0.32 (-3.56)	0.93	0.68
5)	LTD9R	-7.04 (-24.36)	0.31 (5.36)	0.05 (0.97)	-0.29 (-4.93)	2.98 (19.23)	-0.28 (-3.59)	0.91	0.46
6)	LGIC9R	-8.76 (-30.91)	0.43 (7.58)	0.08 (1.45)	-0.38 (-6.53)	3.97 (26.07)	-0.26 (-3.46)	0.95	0.67
7)	LMD9R	-6.92 (-30.52)	0.23 (5.19)	0.03 (0.69)	-0.25 (-5.26)	2.45 (20.13)	-0.22 (-3.62)	0.92	0.51

TABLE XXI(a) (continued)

		Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW
8)	LMT9R	-5.78 (-31.12)	0.24 (6.47)	0.03 (0.95)	-0.25 (-6.56)	2.48 (24.88)	-0.10 (-2.13)	0.95	0.55
9)	LMI9R	0.45 (8.57)	-0.01 (-0.97)	-0.04 (-3.53)	-0.07 (-6.84)	0.59 (20.87)	-0.07 (-5.22)	0.93	0.71
10)	LM29R	0.99 (17.48)	0.05 (4.17)	0.00 (0.21)	-0.05 (-4.61)	0.70 (22.95)	-0.03 (-1.77)	0.95	0.45
11)	LM39R	0.51 (6.66)	0.07 (4.84)	0.01 (0.66)	0.08 (-4.87)	0.96 (23.09)	-0.04 (-2.07)	0.95	0.47

TABLE XXI(b)

BASIC SPECIFICATION: LAGGED ADJUSTMENT: 1955:II - 1968:IV

		Constant	S_1	S_2	S_3	LY9R	LTBR	Lagged Dependent Variable	$\bar{R}^2$	DW
1)	LC9R	-0.21 (-4.12)	-0.04 (-5.18)	-0.01 (-2.10)	-0.03 (-2.34)	0.20 (4.71)	-0.01 (-1.10)	0.56 (5.40)	0.97	2.01
2)	LD9R	-0.01 (-2.10)	-0.04 (-4.17)	-0.01 (-0.58)	-0.03 (-2.23)	0.33 (6.53)	-0.08 (-6.11)	0.59 (7.55)	0.95	2.10
3)	LPN9R	-0.10 (-2.18)	0.06 (8.03)	0.04 (5.29)	0.04 (4.02)	0.13 (2.19)	-0.03 (-2.50)	0.91 (11.28)	0.98	2.10
4)	LON9R	-1.69 (-3.03)	0.13 (3.64)	0.08 (2.70)	-0.02 (-0.61)	0.80 (3.15)	-0.11 (-2.57)	0.84 (12.42)	0.99	1.71
5)	LTD9R	-0.20 (-0.66)	0.05 (2.38)	0.02 (1.01)	0.01 (.52)	0.14 (1.07)	-0.09 (-3.64)	0.98 (23.57)	0.99	1.70
6)	LGIC9R	-0.87 (-1.81)	0.11 (4.02)	0.02 (0.87)	-0.00 (-0.12)	0.42 (1.96)	-0.08 (-2.38)	0.91 (16.93)	0.99	1.67
7)	LMD9R	-0.85 (-2.70)	0.04 (2.19)	0.01 (0.86)	-0.02 (-0.92)	0.38 (3.38)	-0.11 (-5.28)	0.91 (19.86)	0.99	1.45

TABLE XXI(b) (continued)

		Constant	S_1	S_2	S_3	LY9R	LTBR	Lagged Dependent Variable	$\bar{R}^2$	DW
8)	LMT9R	-0.00 (-0.01)	-0.00 (-0.22)	0.00 (0.19)	0.01 (0.69)	0.02 (0.21)	-0.02 (-1.23)	0.99 (26.56)	0.99	2.29
9)	LM19R	0.13 (2.15)	-0.04 (-4.65)	-0.01 (-1.03)	-0.03 (-2.41)	0.30 (6.16)	-0.06 (-5.48)	0.58 (6.73)	0.96	2.10
10)	LM29R	0.11 (1.05)	0.02 (1.99)	0.02 (3.12)	0.02 (1.46)	0.20 (3.50)	-0.04 (-3.66)	0.79 (9.07)	0.98	2.05
11)	LM39R	-0.07 (-1.12)	0.02 (2.45)	0.03 (3.90)	0.02 (1.79)	0.20 (2.92)	-0.04 (-3.82)	0.87 (11.76)	0.99	2.17

TABLE XXI(c)

BASIC SPECIFICATION; AUTOREGRESSIVE TRANSFORMATION: 1955:II - 1968:IV

	Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW	RHO	"Net" $\bar{R}^2$ †
1) LC9R	-0.44 (-5.81)	-0.01 (-2.25)	-0.03 (-6.86)	-0.05 (-10.41)	0.42 (12.31)	0.004 (-0.34)	0.97	2.13	0.70 (6.29)	0.87
2) LD9R	0.01 (0.08)	-0.01 (-1.34)	-0.04 (-4.27)	-0.07 (-7.55)	0.62 (10.04)	-0.07 (-3.22)	0.94	1.99	0.69 (6.46)	0.79
3) LPN9R	0.96 (4.95)	0.05 (6.16)	0.03 (5.11)	0.003 (0.30)	0.37 (4.86)	0.003 (0.27)	0.99	1.98	1.04 (33.68)	0.69
4) LON9R	-2.41 (-1.59)	0.06 (1.34)	0.04 (1.79)	0.001 (0.02)	0.47 (1.25)	-0.06 (-1.09)	0.98	1.63	1.02 (49.32)	0.09
5) LTD9R	1.17 (0.25)	0.04 (2.00)	0.01 (1.04)	-0.02 (-0.83)	0.36 (1.79)	-0.08 (-2.39)	0.99	1.57	0.99 (73.79)	0.15
6) LGIC9R	6.17 (0.42)	0.08 (3.49)	0.02 (1.30)	-0.03 (-1.24)	0.49 (2.33)	-0.17 (-5.23)	0.99	2.19	0.99 (97.50)	0.48
7) LMD9R	-7.13 (-0.26)	0.04 (1.98)	0.005 (0.39)	-0.04 (-1.74)	0.54 (2.66)	-0.02 (-0.54)	0.99	1.22	1.00 (51.92)	0.11

TABLE XXI(c) (continued)

	Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW	RHO	"Net"† $\bar{R}^2$
8) LMT9R	1.44	0.03	-0.004	-0.04	0.37	-0.03	0.99	2.25	1.00	0.25
	(0.56)	(2.54)	(-0.60)	(-3.37)	(3.87)	(-2.12)			(128.82)	
9) LM19R	0.49	-0.01	-0.03	-0.07	0.56	-0.05	0.96	1.99	0.68	0.83
	(4.77)	(-1.65)	(-5.15)	(-8.88)	(11.88)	(-2.91)			(6.13)	
10) LM29R	2.03	0.02	-0.001	-0.02	0.41	-0.02	0.98	1.98	0.99	0.62
	(5.63)	(2.51)	(-0.13)	(-2.65)	(5.24)	(-1.54)			(n.a.)	
11) LM39R	1.59	0.02	0.003	-0.02	0.37	-0.02	0.99	2.02	1.03	0.55
	(7.01)	(2.07)	(0.50)	(-1.61)	(4.23)	(-1.44)			(38.39)	

† "Net" $\bar{R}^2$ is net of RHO but not of seasonality.

TABLE XXI(d)

BASIC SPECIFICATION; TRANSFORMED BY AUTOREGRESSIVE COEFFICIENT: 1955:II - 1968:IV

		Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW	Auto-Regressive Coefficient
1)	LC9R	-0.10 (-4.80)	-0.05 (-8.98)	-0.06 (-9.04)	-0.07 (-8.89)	0.42 (12.83)	-0.004 (-0.33)	0.87	2.10	0.69
2)	LD9R	0.05 (1.28)	-0.06 (-5.95)	-0.08 (-6.34)	-0.10 (-6.96)	0.62 (11.10)	-0.072 (-3.48)	0.80	1.90	0.65
3)	LPN9R	0.10 (5.46)	0.06 (6.93)	-0.06 (-4.15)	-0.07 (-4.03)	0.62 (8.43)	0.005 (0.35)	0.71	1.51	0.86
4)	LON9R	-2.94 (-16.76)	0.17 (3.78)	-0.27 (-5.49)	-0.52 (-8.84)	3.43 (15.56)	-0.091 (-0.10)	0.84	1.85	0.60
5)	LTD9R	-0.86 (-9.59)	0.06 (1.99)	0.25 (-5.79)	-0.34 (-6.48)	1.90 (8.45)	-0.064 (-1.08)	0.56	1.41	0.79
6)	LGIC9R	-2.25 (-14.84)	0.15 (3.74)	-0.38 (-7.76)	-0.57 (-9.72)	3.32 (14.49)	-0.141 (-1.74)	0.80	1.92	0.67
7)	LMD9R	-1.50 (-17.14)	0.05 (2.01)	-0.23 (-6.96)	-0.33 (-8.30)	1.91 (11.83)	0.004 (0.08)	0.73	1.58	0.73

TABLE XXI(d) (continued)

		Constant	S_1	S_2	S_3	LY9R	LTBR	$\bar{R}^2$	DW	Auto-Regressive Coefficient
8)	LMT9R	-1.03	0.04	-0.24	-0.34	1.87	-0.018	0.74	1.59	0.74
		(-13.20)	(1.60)	(-8.08)	(-9.37)	(12.38)	(-0.40)			
9)	LM19R	0.21	-0.06	-0.07	-0.09	0.56	-0.050	0.84	1.94	0.65
		(6.93)	(-7.01)	(-7.48)	(-8.03)	(12.66)	(-3.10)			
10)	LM29R	0.27	0.07	-0.07	-0.08	0.63	-0.014	0.76	1.79	0.79
		(11.91)	(0.92)	(-6.35)	(-6.54)	(11.43)	(-0.96)			
11)	LM39R	0.22	0.02	-0.09	-0.12	0.85	-0.009	0.77	1.78	0.76
		(6.69)	(1.77)	(-6.57)	(-7.43)	(12.45)	(-0.47)			

TABLE XXII

INCLUSION OF INTEREST EXPECTATIONS: 1955:I - 1968:IV

	LY9R	LTBR	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
1) LC9R	0.36	0.01	-0.007	-0.003	0.000	0.003	0.005	0.006	0.007	0.007	0.006	0.005	0.003
$\bar{R}^2$=0.96	(15.97)	(1.18)	(0.004)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)
DW=0.90													
2) LD9R	0.72	-0.06	-0.040	-0.029	-0.020	-0.012	-0.005	0.000	0.004	0.006	0.007	0.006	0.004
$\bar{R}^2$=0.95	(20.50)	(-4.18)	(0.006)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
DW=1.17													
3) LPN9R	0.67	0.03	-0.020	-0.011	-0.003	0.003	0.008	0.012	0.013	0.014	0.013	0.010	0.006
$\bar{R}^2$=0.96	(15.90)	(1.65)	(0.007)	(0.005)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
DW=0.44													
4) LON9R	4.11	-0.15	-0.143	-0.107	-0.074	-0.047	-0.025	-0.007	0.006	0.015	0.018	0.017	0.011
$\bar{R}^2$=0.95	(19.60)	(-1.74)	(0.035)	(0.025)	(0.017)	(0.013)	(0.012)	(0.013)	(0.014)	(0.014)	(0.013)	(0.010)	(0.006)
DW=0.95													
5) LTD9R	2.86	-0.17	-0.091	-0.059	-0.031	-0.009	0.009	0.022	0.030	0.034	0.033	0.027	0.016
$\bar{R}^2$=0.92	(14.59)	(-2.13)	(0.033)	(0.023)	(0.016)	(0.012)	(0.011)	(0.012)	(0.013)	(0.013)	(0.012)	(0.010)	(0.006)
DW=0.46													
6) LGIC9R	3.38	-0.23	-0.034	-0.006	0.018	0.036	0.049	0.057	0.060	0.058	0.051	0.039	0.022
$\bar{R}^2$=0.97	(20.91)	(-3.38)	(0.027)	(0.019)	(0.013)	(0.010)	(0.009)	(0.010)	(0.011)	(0.011)	(0.010)	(0.008)	(0.005)
DW=0.88													
7) LMD9R	2.78	-0.10	-0.100	-0.079	-0.059	-0.043	-0.029	-0.017	-0.008	-0.001	0.003	0.004	0.003
$\bar{R}^2$=0.95	(21.31	(-1.89)	(0.022)	(0.015)	(0.011)	(0.008)	(0.007)	(0.008)	(0.009)	(0.009)	(0.008)	(0.006)	(0.004)
DW=0.82													

TABLE XXII (continued)

	LY9R	LTBR	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
8) LMT9R	2.13	-0.08	-0.023	-0.005	0.009	0.021	0.029	0.034	0.036	0.035	0.031	0.024	0.014
$\overline{R}^2$=0.97	(19.10)	(-1.77)	(0.019)	(0.013)	(0.009)	(0.007)	(0.006)	(0.007)	(0.007)	(0.008)	(0.007)	(0.005)	(0.003)
DW-0.64													
9) LM19R	0.61	-0.04	-0.030	-0.021	-0.014	-0.007	-0.002	0.002	0.005	0.006	0.006	0.006	0.003
$\overline{R}^2$=0.96	(20.92)	(-3.21)	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
DW=1.09													
10) LM29R	0.65	0.00	-0.025	-0.015	-0.008	-0.001	0.004	0.007	0.010	0.010	0.010	0.008	0.005
$\overline{R}^2$=0.96	(19.15)	(0.07)	(0.006)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
DW=0.59													
11) LM39R	0.94	-0.01	-0.031	-0.021	-0.012	-0.005	0.001	0.005	0.008	0.009	0.009	0.008	0.004
$\overline{R}^2$=0.96	(18.80)	(-0.34)	(0.008)	(0.006)	(0.004)	(0.003)	(0.005)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
DW=0.56													

TABLE XXIII

INCLUSION OF INTEREST AND INCOME EXPECTATIONS: 1955:I - 1968:IV

	LTBR / LY9R	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
1) LC9R	0.02	-0.009	-0.007	-0.005	-0.003	-0.002	-0.001	0.000	0.001	0.001	0.001	0.001
$\overline{R}^2=0.97$	(1.87)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
DW-1.68	-0.012	0.010	0.028	0.042	0.052	0.058	0.061	0.060	0.056	0.047	0.035	0.019
	(0.037)	(0.024)	(0.013)	(0.004)	(0.005)	(0.010)	(0.014)	(0.016)	(0.017)	(0.015)	(0.012)	(0.007)
2) LD9R	-0.06	-0.049	-0.037	-0.028	-0.019	-0.012	-0.006	-0.002	0.001	0.003	0.003	0.002
$\overline{R}^2=0.94$	(-3.61)	(0.006)	(0.005)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.002)
DW=1.24	0.148	0.130	0.114	0.098	0.083	0.070	0.057	0.045	0.034	0.024	0.015	0.007
	(0.065)	(0.042)	(0.023)	(0.008)	(0.009)	(0.018)	(0.025)	(0.029)	(0.029)	(0.027)	(0.021)	(0.012)
3) LPN9R	0.04	-0.024	-0.018	-0.013	-0.008	-0.004	-0.001	0.001	0.003	0.003	0.003	0.002
$\overline{R}^2=0.97$	(2.35)	(0.007)	(0.005)	(0.004)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.002)
DW=0.68	-0.015	0.023	0.055	0.079	0.097	0.108	0.113	0.111	0.102	0.087	0.064	0.036
	(0.070)	(0.045)	(0.025)	(0.008)	(0.009)	(0.019)	(0.027)	(0.030)	(0.031)	(0.028)	(0.022)	(0.013)
4) LON9R	-0.10	-0.169	-0.154	-0.139	-0.124	-0.109	-0.093	-0.078	-0.063	-0.047	-0.031	-0.016
$\overline{R}^2=0.98$	(-2.06)	(0.021)	(0.015)	(0.011)	(0.010)	(0.011)	(0.012)	(0.013)	(0.012)	(0.011)	(0.009)	(0.005)
DW=0.96	-0.191	0.081	0.307	0.486	0.619	0.704	0.744	0.736	0.682	0.582	0.434	0.241
	(0.213)	(0.139)	(0.075)	(0.025)	(0.029)	(0.059)	(0.081)	(0.094)	(0.096)	(0.087)	(0.069)	(0.039)
5) LTD9R	-0.21	-0.160	-0.095	-0.039	0.006	0.041	0.066	0.080	0.085	0.079	0.063	0.037
$\overline{R}^2=0.94$	(-3.02)	(0.029)	(0.020)	(0.015)	(0.014)	(0.015)	(0.017)	(0.018)	(0.017)	(0.015)	(0.012)	(0.007)
DW=0.38	2.187	1.551	0.998	0.527	0.139	-0.167	-0.391	-0.532	-0.590	-0.566	-0.460	-0.271
	(0.296)	0.193	0.105	(0.035)	(0.040)	(0.082)	(0.113)	(0.130)	(0.133)	(0.121)	(0.095)	(0.055)
6) LGIC9R	-0.24	-0.095	-0.046	-0.005	0.027	0.052	0.069	0.077	0.078	0.071	0.055	0.032
$\overline{R}^2=0.97$	(-3.71)	(0.027)	(0.019)	(0.014)	(0.013)	(0.014)	(0.015)	(0.016)	(0.016)	(0.014)	(0.011)	(0.006)
DW=0.32	1.664	1.239	0.866	0.545	0.276	0.059	-0.105	-0.218	-0.279	-0.287	-0.243	-0.148
	(0.274)	(0.178)	(0.097)	(0.032)	(0.037)	(0.076)	(0.104)	(0.120)	(0.123)	(0.112)	(0.088)	(0.051)

TABLE XXIII (continued)

INCLUSION OF INTEREST AND INCOME EXPECTATIONS: 1955:I - 1968:IV

	LTBR / LY9R	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
7) LMD9R	-0.11	-0.149	-0.112	-0.081	-0.054	-0.031	-0.013	0.000	0.009	0.014	0.014	0.009
$\bar{R}^2$=0.96	(-2.34)	(0.020)	(0.014)	(0.010)	(0.010)	(0.010)	(0.012)	(0.012)	(0.012)	(0.011)	(0.008)	(0.005)
DW=0.29	1.272	0.958	0.682	0.443	0.243	0.080	-0.045	-0.132	-0.181	-0.193	-0.166	-0.102
	(0.206)	(0.134)	(0.073)	(0.024)	(0.028)	(0.057)	(0.079)	(0.090)	(0.092)	(0.084)	(0.066)	(0.038)
8) LMT9R	-0.10	-0.066	-0.031	-0.002	0.022	0.040	0.051	0.058	0.058	0.052	0.041	0.023
$\bar{R}^2$=0.97	(-2.30)	(0.018)	(0.012)	(0.009)	(0.008)	(0.009)	(0.010)	(0.011)	(0.010)	(0.009)	(0.007)	(0.004)
DW=0.33	1.272	0.925	0.622	0.362	0.146	-0.025	-0.153	-0.237	-0.278	-0.274	-0.227	-0.135
	(0.180)	(0.117)	(0.064)	(0.021)	(0.024)	(0.050)	(0.069)	(0.079)	(0.081)	(0.074)	(0.058)	(0.033)
9) LM19R	-0.03	-0.036	-0.028	-0.020	-0.014	-0.009	-0.004	-0.001	0.001	0.002	0.003	0.002
$\bar{R}^2$=0.95	(-2.65)	(0.005)	(0.004)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
DW=1.32	0.097	0.092	0.087	0.080	0.074	0.066	0.059	0.050	0.041	0.032	0.022	0.011
	(0.053)	(0.035)	(0.019)	(0.006)	(0.007)	(0.015)	(0.020)	(0.023)	(0.024)	(0.022)	(0.017)	(0.010)
10) LM29R	0.01	-0.030	-0.022	-0.016	-0.011	-0.007	-0.003	-0.000	0.001	0.002	0.002	0.002
$\bar{R}^2$=0.97	(0.68)	(0.006)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
DW=0.92	0.024	0.047	0.066	0.080	0.089	0.094	0.094	0.090	0.081	0.067	0.049	0.027
	(0.057)	(0.037)	(0.020)	(0.007)	(0.008)	(0.016)	(0.022)	(0.025)	(0.025)	(0.023)	(0.018)	(0.010)
11) LM39R	0.01	-0.036	-0.031	-0.027	-0.023	-0.019	-0.016	-0.013	-0.010	-0.007	-0.004	-0.002
$\bar{R}^2$=0.98	(0.46)	(-0.006)	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.001)
DW=1.00	--0.080	-0.006	0.057	0.107	0.145	0.170	0.183	0.183	0.171	0.147	0.111	0.061
	(0.063)	(0.041)	(0.022)	(0.007)	(0.008)	(0.017)	(0.024)	(0.028)	(0.028)	(0.026)	(0.020)	(0.012)

TABLE XXIV

INCLUSION OF INTEREST, INCOME AND PRICE EXPECTATIONS: 1955:I - 1968:IV

	LTBR / LY9R / SAPDOT	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
1) LC9R	0.02	-0.005	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.004	-0.003	-0.002	-0.001
$\bar{R}^2=0.99$	(3.30)	(0.003)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
DW=1.07	-0.025	0.003	0.026	0.045	0.059	0.068	0.072	0.072	0.067	0.057	0.043	0.024
	(0.026)	(0.017)	(0.009)	(0.003)	(0.004)	(0.008)	(0.010)	(0.012)	(0.012)	(0.011)	(0.009)	(0.005)
	-0.924	-0.685	-0.476	-0.296	-0.145	-0.024	-0.068	0.130	0.163	0.166	0.140	0.085
	(0.128)	(0.104)	(0.090)	(0.086)	(0.086)	(0.088)	(0.088)	(0.085)	(0.078)	(0.067)	(0.050)	(0.028)
2) LD9R	-0.05	-0.041	-0.034	-0.028	-0.022	-0.017	-0.012	-0.009	-0.006	-0.003	-0.002	-0.001
$\bar{R}^2=0.96$	(-3.68)	(0.006)	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.002)
DW=0.96	0.117	0.113	0.108	0.102	0.095	0.086	0.077	0.067	0.056	0.043	0.030	0.015
	(0.059)	(0.038)	(0.021)	(0.068)	(0.084)	(0.017)	(0.023)	(0.026)	(0.027)	(0.024)	(0.019)	(0.011)
	-1.142	-0.917	-0.715	-0.537	-0.382	-0.252	-0.145	-0.061	-0.002	0.034	0.047	0.035
	(0.287)	(0.233)	(0.203)	(0.192)	(0.193)	(0.197)	(0.198)	(0.191)	(0.176)	(0.149)	(0.111)	(0.062)
3) LPN9R	0.05	-0.012	-0.012	-0.012	-0.011	-0.10	-0.10	-0.008	-0.007	-0.006	-0.004	-0.002
$\bar{R}^2=0.98$	(3.49)	(0.006)	(0.004)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)	(0.001)
DW-0.49	-0.069	-0.008	0.042	0.083	0.114	0.134	0.145	0.146	0.137	0.117	0.088	0.049
	(0.057)	(0.037)	(0.020)	(0.007)	(0.008)	(0.016)	(0.022)	(0.026)	(0.026)	(0.024)	(0.019)	(0.011)
	-1.397	-1.199	-1.016	-0.847	-0.694	-0.555	-0.431	-0.322	-0.228	-0.149	-0.084	-0.035
	(0.297)	(0.227)	(0.197)	(0.187)	(0.188)	(0.192)	(0.193)	(0.186)	(0.171)	(0.145)	(0.108)	(0.060)
4) LON9R	-0.08	-0.137	-0.138	-0.136	-0.132	-0.124	-0.114	-0.102	-0.087	-0.069	-0.049	-0.026
$\bar{R}^2=0.99$	(-1.88)	(0.021)	(0.014)	(0.010)	(0.009)	(0.010)	(0.012)	(0.013)	(0.013)	(0.011)	(0.009)	(0.005)
DW=0.93	-0.333	-0.020	0.274	0.495	0.660	0.771	0.826	0.826	0.771	0.661	0.496	0.276
	(0.195)	(0.126)	(0.068)	(0.023)	(0.028)	(0.056)	(0.076)	(0.087)	(0.089)	(0.081)	(0.063)	(0.036)
	-3.372	-2.947	-2.548	-2.176	-1.830	-1.509	-1.215	-0.947	-0.706	-0.490	-0.301	-0.137
	(0.947)	(0.770)	(0.669)	(0.633)	(0.637)	(0.651)	(0.653)	(0.632)	(0.579)	(0.492)	(0.367)	(0.204)

TABLE XXIV (continued)

	LTBR LY9R SAPDOT	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
5) LTD9R	-0.19	-0.090	-0.049	-0.014	0.014	0.035	0.050	0.058	0.060	0.055	0.043	0.025
$\bar{R}^2$=0.98	(-4.27)	(0.020)	(0.014)	(0.010)	(0.009)	(0.010)	(0.012)	(0.012)	(0.012)	(0.011)	(0.009)	(0.005)
DW=0.58	1.818	1.320	0.885	0.514	0.205	-0.041	-0.224	-0.344	-0.401	-0.395	-0.326	-0.195
	(0.189)	(0.123)	(0.066)	(0.022)	(0.027)	(0.054)	(0.074)	(0.085)	(0.086)	(0.079)	(0.062)	(0.035)
	-0.594	-2.158	-3.429	-4.407	-5.091	-5.481	-5.579	-5.382	-4.893	-4.110	-3.033	-1.663
	(0.920)	(0.748)	(0.650)	(0.615)	(0.619)	(0.632)	(0.635)	(0.614)	(0.563)	(0.478)	(0.357)	(0.198)
6) LGIC9R	-0.21	-0.016	0.003	0.018	0.031	0.039	0.044	0.046	0.044	0.038	0.029	0.016
$\bar{R}^2$=0.99	(-6.59)	(0.015)	(0.010)	(0.007)	(0.006)	(0.007)	(0.008)	(0.009)	(0.009)	(0.008)	(0.006)	(0.004)
DW=1.39	1.262	0.990	0.749	0.537	0.356	0.206	0.085	-0.005	-0.064	-0.094	-0.093	-0.062
	(0.137)	(0.089)	(0.048)	(0.016)	(0.019)	(0.039)	(0.053)	(0.061)	(0.062)	(0.057)	(0.045)	(0.026)
	-2.157	-3.370	-4.329	-5.035	-5.488	-5.688	-5.634	-5.328	-4.769	-3.956	-2.891	-1.572
	(0.666)	(0.541)	(0.470)	(0.446)	(0.448)	(0.458)	(0.459)	(0.444)	(0.408)	(0.346)	(0.258)	(0.143)
7) LMD9R	-0.10	-0.115	-0.091	-0.070	-0.051	-0.036	-0.023	-0.012	-0.004	0.001	0.003	0.003
$\bar{R}^2$=0.97	(-2.37)	(0.020)	(0.013)	(0.010)	(0.009)	(0.010)	(0.011)	(0.012)	(0.012)	(0.011)	(0.008)	(0.005)
DW=0.31	1.096	0.849	0.629	0.439	0.276	0.142	0.037	-0.041	-0.089	-0.110	-0.101	-0.065
	(0.185)	(0.120)	(0.065)	(0.021)	(0.026)	(0.053)	(0.072)	(0.083)	(0.084)	(0.077)	(0.060)	(0.035)
	-0.654	-1.279	-1.781	-2.159	-2.413	-2.544	-2.551	-2.435	-2.195	-1.831	-1.344	-0.734
	(0.899)	(0.730)	(0.634)	(0.601)	(0.605)	(0.618)	(0.620)	(0.599)	(0.550)	(0.467)	(0.349)	(0.193)
8) LMT9R	-0.08	-0.013	0.002	0.015	0.024	0.031	0.035	0.036	0.035	0.030	0.023	0.013
$\bar{R}^2$=0.99	(-4.13)	(0.009)	(0.006)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.002)
DW=1.17	1.001	0.757	0.542	0.356	0.200	0.073	-0.025	-0.094	-0.134	-0.144	-0.125	-0.077
	(0.080)	(0.052)	(0.028)	(0.009)	(0.011)	(0.023)	(0.031)	(0.036)	(0.037)	(0.033)	(0.026)	(0.015)
	-1.365	-2.213	-2.885	-3.383	-3.707	-3.855	-3.829	-3.627	-3.252	-2.701	-1.975	-1.075
	(0.391)	(0.317)	(0.276)	(0.261)	(0.263)	(0.268)	(0.269)	(0.261)	(0.239)	(0.203)	(0.152)	(0.084)

		LTBR / LY9R / SAPDOT	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11
9)	LM19R	-0.03	-0.030	-0.025	-0.021	-0.017	-0.014	-0.010	-0.008	-0.005	-0.003	-0.002	-0.001
	$\overline{R}^2$=0.97	(-2.72)	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
	DW=0.91	0.072	0.078	0.082	0.084	0.084	0.081	0.076	0.069	0.060	0.048	0.034	0.018
		(0.045)	(0.029)	(0.016)	(0.005)	(0.006)	(0.013)	(0.018)	(0.020)	(0.021)	(0.019)	(0.015)	(0.008)
		-1.081	-0.849	-0.643	-0.462	-0.308	-0.179	-0.076	0.001	0.052	0.078	0.078	0.052
		(0.220)	(0.179)	(0.155)	(0.147)	(0.148)	(0.151)	(0.151)	(0.147)	(0.134)	(0.114)	(0.085)	(0.047)
10)	LM29R	0.01	-0.027	-0.028	-0.028	-0.027	-0.026	-0.024	-0.021	-0.018	-0.015	-0.010	-0.005
	$\overline{R}^2$=0.99	(1.17)	(0.005)	(0.004)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
	DW=0.61	-0.115	-0.025	0.051	0.112	0.158	0.190	0.207	0.209	0.197	0.169	0.128	0.071
		(0.050)	(0.032)	(0.017)	(0.006)	(0.007)	(0.014)	(0.019)	(0.022)	(0.023)	(0.021)	(0.016)	(0.009)
		-1.425	-1.127	-0.861	-0.627	-0.427	-0.259	-0.124	-0.022	0.048	0.085	0.089	0.061
		(0.241)	(0.196)	(0.170)	(0.161)	(1.62)	(0.166)	(0.166)	(0.161)	(0.147)	(0.125)	(0.094)	(0.052)
11)	LM39R	0.02	-0.020	-0.018	-0.016	-0.014	-0.012	-0.011	-0.009	-0.007	-0.005	-0.003	-0.002
	$\overline{R}^2$=0.98	(1.53)	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.001)
	DW=0.58	-0.018	0.023	0.057	0.083	0.103	0.115	0.120	0.118	0.109	0.092	0.069	0.038
		(0.004)	(0.029)	(0.016)	(0.005)	(0.006)	(0.013)	(0.017)	(0.020)	(0.020)	(0.018)	(0.014)	(0.008)
		-1.276	-1.060	-0.865	-0.689	-0.533	-0.397	-0.281	-0.184	-0.108	-0.051	-0.014	-0.003
		(0.216)	(0.176)	(0.153)	(0.145)	(0.145)	(0.149)	(0.149)	(0.144)	(0.132)	(0.112)	(0.084)	(0.047)